

Rivelin Primary School



Calculation Policy

Reviewed: July 2024

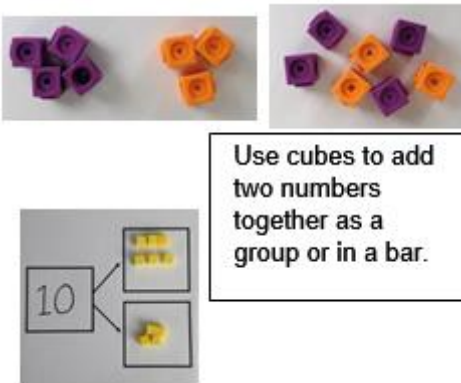
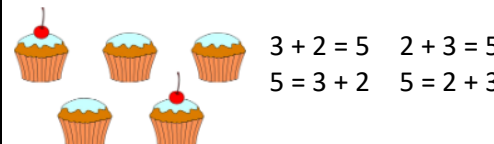
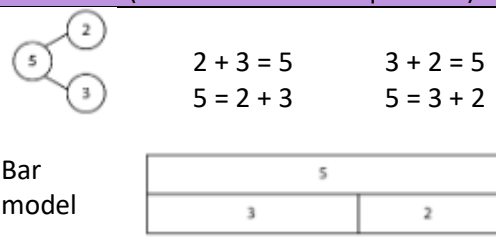
Next Review: December 2024

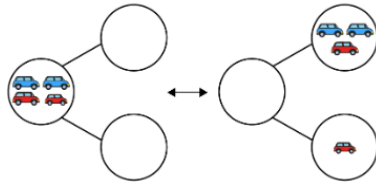
CALCULATION POLICY

This policy lays out the expectations for both mental and written calculations for the four number operations and has been created to support the teaching of a mastery approach to mathematics. This is underpinned by the use of models and images that support conceptual understanding and this policy promotes a range of representations to be used across the primary years. Mathematical understanding is developed through use of representations that are first of all concrete (e.g. counters and multilink cubes), and then pictorial (e.g. part whole) to then facilitate abstract working (e.g. formal written methods). This policy is a guide through an appropriate progression of representations and if at any point a pupil is struggling with the abstract, they should revert to familiar pictorial and/or concrete materials/representations as appropriate. As children move through the different stages, representations should be modelled alongside each other to ensure a secure understanding is maintained. Children should only move onto the abstract method when they have a secure understanding of the two former methods.

Although this policy sets out the main methods of mental and written calculations to be taught, it has been appended with a list of recommendations and effective practice teaching ideas aimed at informing and enhancing teaching across all the primary phases. Many of these models and images come from the White Rose materials. Some models are also supplemented by the NCETM's Spine materials and Mastering Number.

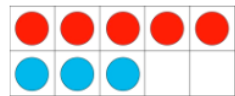
Addition

Stem sentences	Concrete (Can we make it?)	Pictorial (Can we draw it?)	Abstract (Can we write the equation?)
<p>___ is the whole, ___ is a part, ___ is a part.</p> <p>___ = ___ plus ___ and ___ plus ___ = ___</p> <p>There are ___ in total.</p> <p>___ is a part.</p> <p>___ + ___ = _____</p> <p>___ = _____ + _____</p> <p>There are ___ red counters and ___ yellow counters.</p> <p>There are _____ counters altogether.</p> <p>This means that _____ and _____ are a bond to _____. OR</p> <p>There are _____ counters altogether.</p> <p>___ + ___ = _____</p> <p>One part is ___ and the other part is ____.</p> <p>The whole is _____.</p> <p>___ plus ___ is equal to _____.</p> <p>___ + ___ = _____</p> <p>___ = _____ + _____</p> <p>FS/Y1</p>	<p><i>*When adding numbers to 10, children can explore both aggregation and augmentation.</i></p> <p>Aggregation – Combining 2 or more parts to make a whole; the addition symbol, +, can be used to represent aggregation.</p> <p><i>The part-whole model, discrete and continuous bar model, number shapes and ten frame support aggregation.</i></p> <p>Augmentation – An addition context described by a ‘first.., then..., now...’ story is an example of this. We can link the story to a numerical representation – each number represents something in the story.</p> <p><i>The combination bar model, ten frame, bead string and number track all support augmentation.</i></p> 	 <p>3 + 2 = 5 2 + 3 = 5</p> <p>5 = 3 + 2 5 = 2 + 3</p>	 <p>2 + 3 = 5 3 + 2 = 5</p> <p>5 = 2 + 3 5 = 3 + 2</p> <p>Bar model</p>



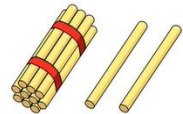
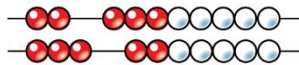
$$3 + 4 = 7 \quad 7 = 3 + 4$$

$$4 + 3 = 7 \quad 7 = 4 + 3$$



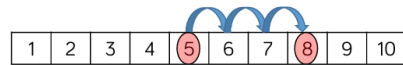
$$5 + 3 = 8 \quad 8 = 5 + 3$$

$$3 + 5 = 8 \quad 8 = 3 + 5$$

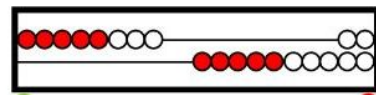


10, 11, 12

$$5 + 3 = 8$$



*Mastering Number Programme



$$5 + 3 = 8$$

First... Then... Now...

e.g. **First** there were 4 children on the bus, **then** 3 children got on. **Now** there are 7 children on the bus.

First there were ____.
 Then ____ more were added.
 Now there are ____.
 ____ + ____ = ____

Counting on

I need to start counting from ____.
 The number that comes after ____ is ____.
 I will say the number ____ because...
 I will not say the number ____ because...

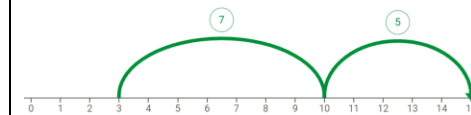
FS/Y1

If I have ____ counters, I need to add ____ more counters to make ____.
 I need to add ____ to ____ to make ____.

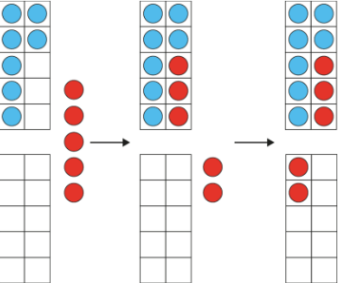
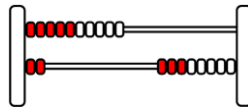
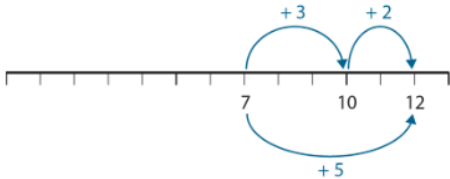
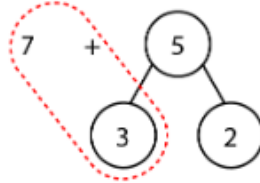
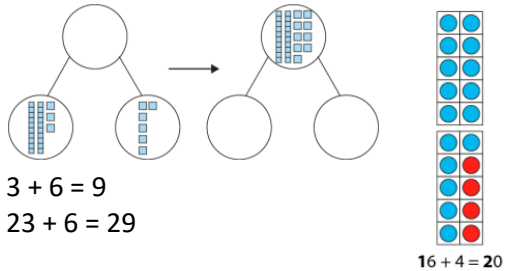
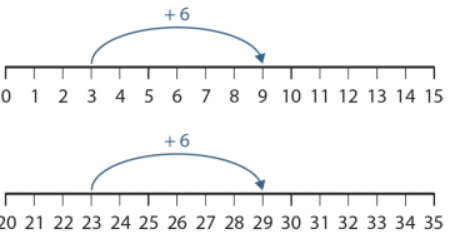
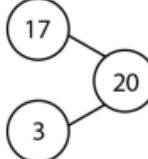
We can look for pairs of addends which sum to 10.
 ____ plus ____ is equal to 10, then 10 plus ____ is equal to ____.

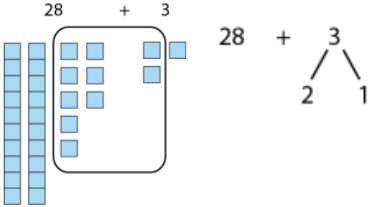
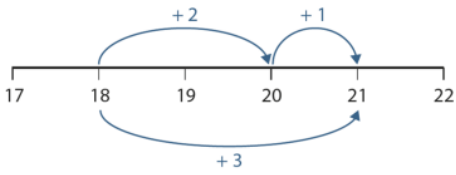
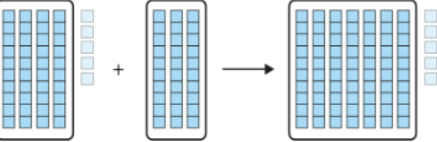

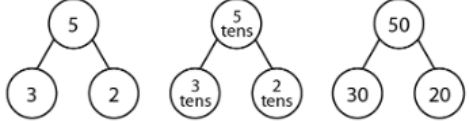
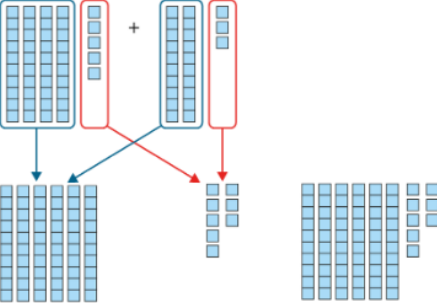
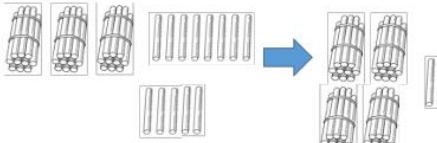
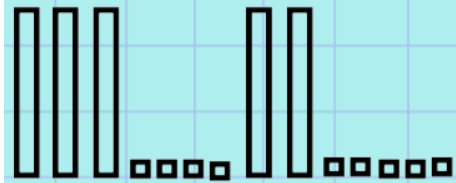
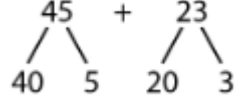
____ ones + ____ ones = ____ ones
 So ____ ones + ____ ones + ____ ones = ____ ones
 ____ and ____ are a bond to ____
 10 + ____ = ____
 So ____ + ____ = ____

Role play getting 'on the bus' or use a toy bus.



$3 + 5 + 7 = 3 + 7 + 5 = 10 + 5 = 15$

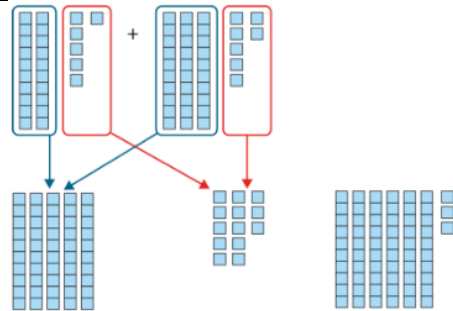
<p>Year 2</p>	<p><i>Manipulatives that highlight number bonds to 10 are effective when adding three 1-digit numbers.</i></p> <p><i>When adding one-digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten.</i></p> <p><i>From Y2, use different manipulatives to represent this exchange alongside number lines to support children in understanding how to partition their jumps.</i></p>		
<p>First, I partition the __: __ plus __ is equal to __.</p> <p>Then __ plus __ is equal to ten ... and ten plus __ is equal to __.</p> <p>Year 2</p> <p><i>When adding single digits to a two-digit number, children should be encouraged to count on from the larger number.</i></p>	 <p>7 + 5 = 7 + 3 = 10 10 + 2 = 12</p> <p><u>*Mastering Number Programme</u></p>  <p>10 + 2 = 12</p>	 <p>7 + 5 = 7 + 3 = 10 10 + 2 = 12</p>	 <p>7 + 3 = 10 10 + 2 = 12</p>
<p>I know that __ plus __ is equal to __. (single-digit fact)</p> <p>So __ plus __ is equal to __. (related two-digit plus single digit fact)</p> <p>I know that __ plus __ is equal to ten so __ plus __ is equal to __.</p> <p>There are __ groups of 10 and __ more.</p> <p>There are __ in total.</p> <p>There are __ tens and __ ones.</p> <p>The number is __.</p>	 <p>3 + 6 = 9 23 + 6 = 29</p> <p>16 + 4 = 20</p>	 <p>3 + 6 = 9</p>	 <p>17 + 3 = 20</p>

<p>Year 2</p>	 <p>$28 + 3$</p> <p>$28 + 3$</p>	 <p>$23 + 6 = 29$</p>	
<p>I know that ___ plus ___ is equal to ___. So ___ tens plus ___ tens is equal to ___ tens.</p> <p>___ tens and ___ ones, plus ___ tens is equal to ___ tens and ___ ones.</p>	 <p>$40 + 30 = 70$ so $45 + 30 = 75$</p>	 <p>$45 + 30 = 75$</p>	 <p>$2 + 3 = 5$ $2 \text{ tens} + 3 \text{ tens} = 5 \text{ tens}$ $20 + 30 = 50$</p>
<p>Year 2</p> <p>First, I partition the ___ into ___ and ___, and the ___ into ___ and ___. ___ plus ___ is equal to ___... (addition of the tens) ___ plus ___ is equal to ___... (addition of the ones) and ___ plus ___ is equal to ___. (addition of the tens and ones) So ___ plus ___ is equal to ___. (summary of the overall calculation)</p>	 <p>$45 + 23 = 60 + 8 = 68$</p>  <p>$30 + 13 \Rightarrow 40 + 3 = 43$</p>	<p><i>Real story</i></p>  <p>$34 + 25 =$</p>	 <p>$45 + 23$</p> <p>$40 + 20 = 60$ $5 + 3 = 8$ $60 + 8 = 68$</p>

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

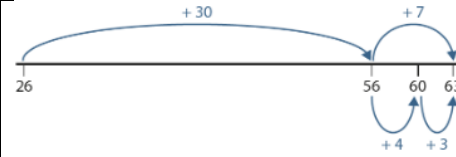
Hundred squares and straws can support children to find the number bond to 10.

First I partition the into and , and the into and .
 plus is equal to ... (addition of the tens)
 plus is equal to ... (addition of the ones)
and plus is equal to . (addition of the tens and ones)
So plus is equal to . (summary of the overall calculation)



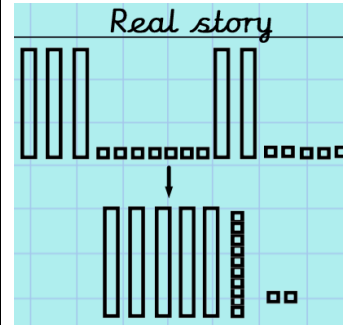
$$26 + 37 = 50 + 13 = 63$$

Year 2

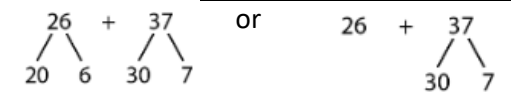


$$26 + 30 = 56$$

$$56 + 7 = 63$$



$$37 + 25 = 62$$



$$20 + 30 = 50$$

$$6 + 7 = 13$$

$$50 + 13 = 63$$

$$26 + 30 = 56$$

$$56 + 7 = 63$$

Addition Facts

Adding 1

Bonds to 10

Adding 10

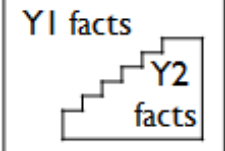
Bridging/compensating

Adding 2

Adding 0

Doubles

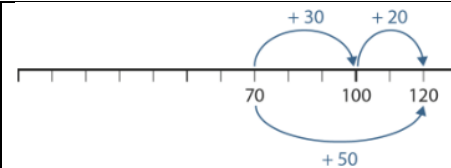
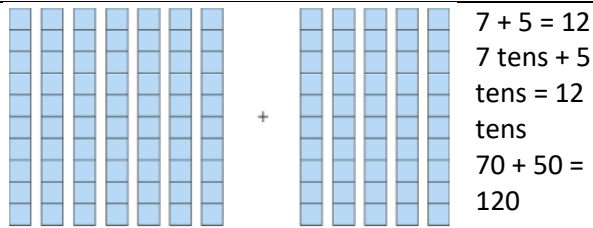
Near doubles



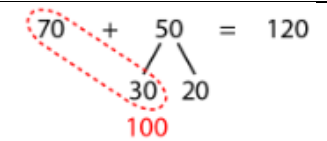
+	0	1	2	3	4	5	6	7	8	9	10
0	0+0	0+1	0+2	0+3	0+4	0+5	0+6	0+7	0+8	0+9	0+10
1	1+0	1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9	1+10
2	2+0	2+1	2+2	2+3	2+4	2+5	2+6	2+7	2+8	2+9	2+10
3	3+0	3+1	3+2	3+3	3+4	3+5	3+6	3+7	3+8	3+9	3+10
4	4+0	4+1	4+2	4+3	4+4	4+5	4+6	4+7	4+8	4+9	4+10
5	5+0	5+1	5+2	5+3	5+4	5+5	5+6	5+7	5+8	5+9	5+10
6	6+0	6+1	6+2	6+3	6+4	6+5	6+6	6+7	6+8	6+9	6+10
7	7+0	7+1	7+2	7+3	7+4	7+5	7+6	7+7	7+8	7+9	7+10
8	8+0	8+1	8+2	8+3	8+4	8+5	8+6	8+7	8+8	8+9	8+10
9	9+0	9+1	9+2	9+3	9+4	9+5	9+6	9+7	9+8	9+9	9+10
10	10+0	10+1	10+2	10+3	10+4	10+5	10+6	10+7	10+8	10+9	10+10

Stem sentences	Concrete (Can we make it?)	Pictorial (Can we draw it?)	Abstract (Can we write the equation?)
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I know that ___ plus ___ is equal to _____. (single-digit addends)
 So ___ tens plus ___ tens is equal to ___ tens. (multiple-of-ten addends)
 ___ plus ___ is equal to one hundred and _____.



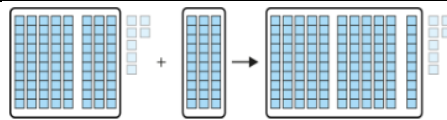
$70 + 50 =$
 $70 + 30 = 100$
 $100 + 20 = 120$



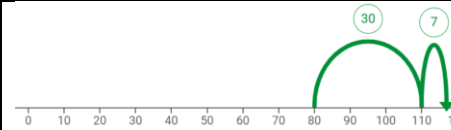
$70 + 50 = 70 + 30 + 20$
 $= 100 + 20$
 $= 120$

Year 3

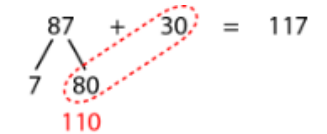
I know that ___ plus ___ is equal to _____. (single-digit addends)
 So ___ tens plus ___ tens is equal to ___ tens. (multiple-of-ten addends)
 ___ plus ___ is equal to one hundred and _____.



$87 + 30 = 110 + 7 = 117$



$87 + 30 = 80 + 30 + 7$
 $= 110 + 7$
 $= 117$



$87 + 30 = 80 + 7 + 30$
 $= 110 + 7$
 $= 117$

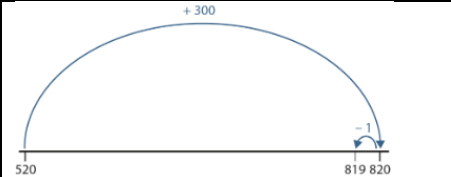
When adding single digits to a two-digit number, children should be encouraged to count on from the larger number.

Children should also apply their knowledge of number bonds to add more efficiently e.g. $8 + 5 = 13$ so $38 + 5 = 43$.

Year 3

First, we add: ___ plus ___ is equal to ___ ...
 ... then we adjust: ___ minus ___ is equal to _____.

$35 + 49 = 34 + 50 = 84$



$520 + 299 =$
 $520 + 300 = 820$
 $820 - 1 = 819$

$69 + 69 = 138$
 $70 + 70 = 140$
 $140 - 2 = 138$

___ ones + ___ ones = ___ ones

If I have ___ ones, I can
exchange them for ___ ten and
___ ones.

I have ___ hundreds, ___ tens
and ___ ones, so altogether I
have ___.

Year 3

We line up the ones; ___ ones plus ___ ones.

We line up the tens: ___ tens plus ___ tens.

The ___ is in the ones column – it represents ___ ones. The ___ is in the ones column – it represents ___ ones.

___ ones plus ___ ones is equal to ___ ones.

The ___ is in the tens column – it represents ___ tens. The ___ is in the tens column – it represents ___ tens.

___ tens plus ___ tens is equal to ___ tens.

In column addition we start at the right-hand side.

___ ones + ___ ones = ___ ones

If I have ___ ones, I can exchange them for ___ ten and ___ ones.

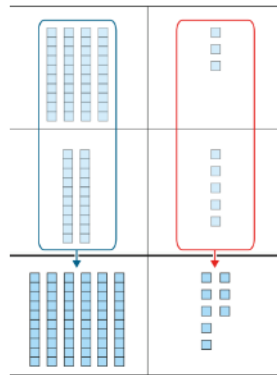
I have ___ hundreds, ___ tens and ___ ones, so altogether I have ___.

From Year 3, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.

When teaching the column method, initially ask pupils to write column headings above their calculation (Th, H, T, O, etc.) and make a point of reminding them that there should be one digit in each square. This will help ensure that pupils line up their columns correctly.

After setting up the columns, pupils will always begin with the smallest place value and then move to the left

Start with two-digit numbers to exemplify lining up the columns.

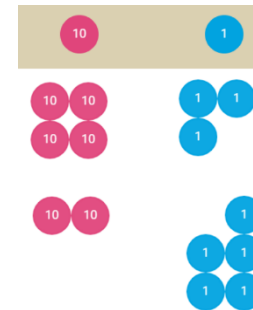


- Dexter uses base 10 to work out $208 + 313$

	Hundreds	Tens	Ones
+			
	5	2	1

	H	T	O
	2	0	8
+	3	1	3
	5	2	1

Children could draw place value counters.



Start with two-digit numbers to exemplify lining up the columns.

	4	3
+	2	5

$$\begin{array}{r} 462 \\ + 205 \\ \hline \end{array}$$

Remind children that when using column method that the bigger number goes on the top. The operation symbol (+, -, x and ÷) need to go to the left of the equation.

Year 3

If the column sum is equal to ten or more, we must exchange.

___ ones + ___ ones = ___ ones

If I have ___ ones, I can exchange them for ___ ten and ___ ones.

I have ___ hundreds, ___ tens and ___ ones, so altogether I have ___.

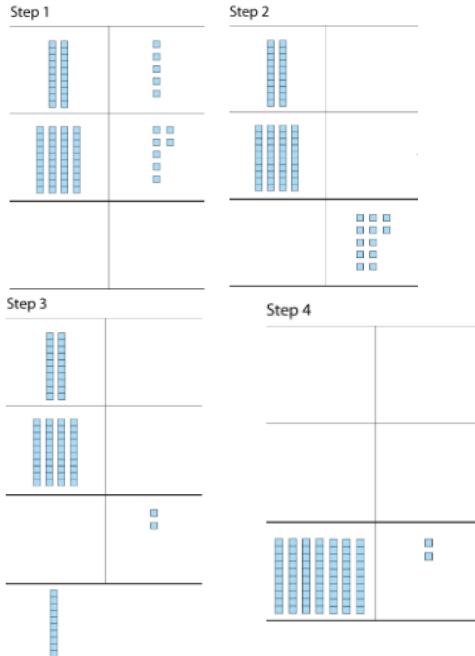
___ tens + ___ tens = ___ tens

If I have ___ tens, I can exchange them for ___ hundred and ___ tens.

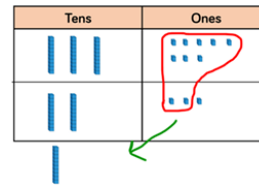
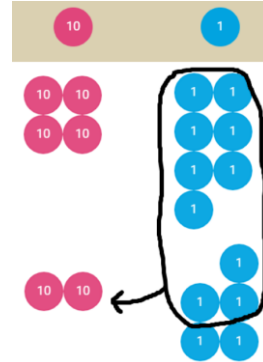
I have ___ hundreds, ___ tens and ___ ones, so altogether I have ___.

**Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 3 digits. Plain counters on a PV grid can also be used to support learning. Plain counters on a place value grid can also be used to support learning.*

Start with two-digit numbers to exemplify the exchanging.



Children could draw place value counters.



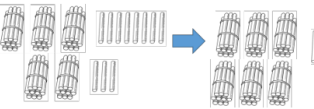
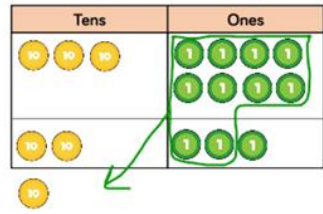
$$\begin{array}{r} 38 \\ + 23 \\ \hline 61 \\ 1 \end{array}$$

Ensure children write out their calculation alongside any concrete resources so they can see links to the written column method.

Start with two-digit numbers to exemplify the exchanging.

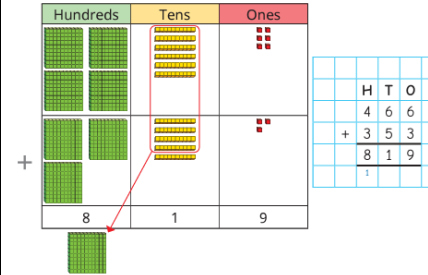
$$\begin{array}{r} 25 \\ + 47 \\ \hline 72 \\ 1 \end{array}$$

$$\begin{array}{r} 567 \\ + 233 \\ \hline 800 \\ 11 \end{array}$$



$$38 + 23 = 61$$

Nijah uses base 10 to work out $466 + 353$



	H	T	O
	4	6	6
+	3	5	3
	8	1	9

If the column sum is equal to ten or more, we must exchange.

Year 4

___ ones added to ___ ones is equal to ___ ones.

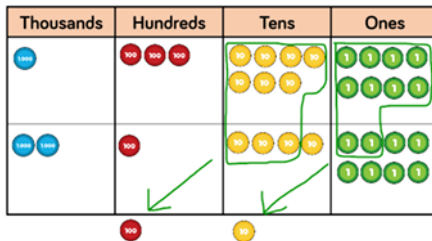
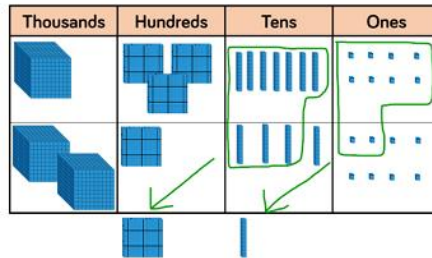
___ plus ___ plus the 1 that I exchanged from the last column is equal to ___.

I have ___ hundreds/tens/ones, so I do/do not need to make an exchange.

I can exchange 10 ___ for 1 ___.

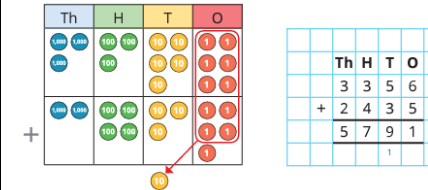
**Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 4 digits. Plain counters on a PV grid can also be used to support learning.*

See Year 3 examples



See Year 3 examples

Kim uses counters to find the total of 3,356 and 2,435



Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method. Plain counters on a place value grid can also be used to support learning.

	1	3	7	8
+	2	1	4	8
	3	5	2	6
			1	1

$$\begin{array}{r}
 6,584 \\
 + 2,739 \\
 \hline
 9,323 \\
 1\ 1\ 1
 \end{array}
 \quad
 \begin{array}{r}
 \pounds 24.55 \\
 + \pounds 17.82 \\
 \hline
 \pounds 42.37 \\
 1\ 1
 \end{array}$$

If the column sum is equal to ten or more, we must exchange.

Years 5 and 6

*Base 10 and place value counters are the most effective manipulatives when adding numbers with more than 4 digits.

At this stage, children should be encouraged to work in the abstract, using the column method to add larger numbers efficiently.

Place value counters and plain counters on a place value grid are the most effective manipulatives when adding decimals with 1, 2 and then 3 decimal places.

Ensure children have experience of adding decimals with a variety of decimal places. This includes putting this into context when adding money and other resources.

See Year 3 examples

TTh	Th	H	T	O
3	9	8	1	5

Adding with up to 3 decimal places

Ones	Tenths	Hundredths
1 1 1	0.1 0.1 0.1	0.01 0.01 0.01
1 1	0.1 0.1 0.1	0.01 0.01
1	0.1	

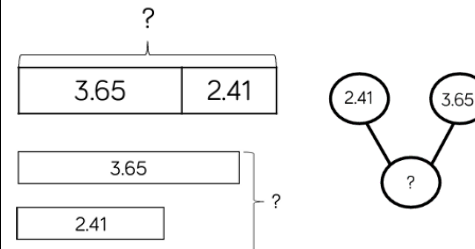
Ones	Tenths	Hundredths
3	6	5
2	4	1

See Year 3 examples

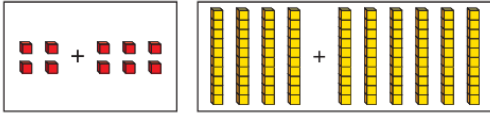
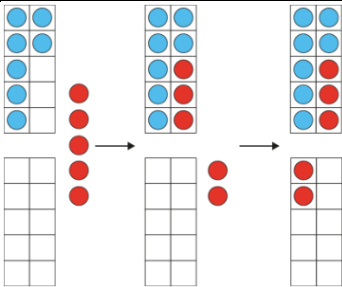
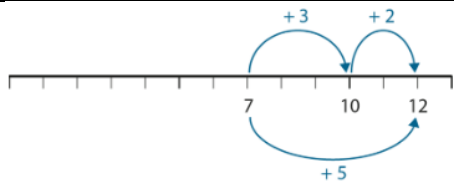
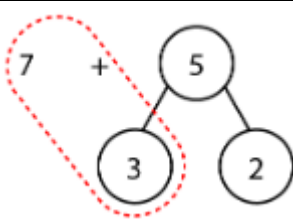

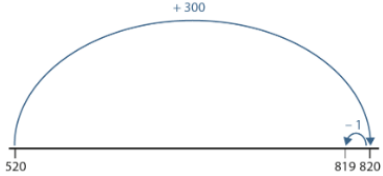
As in Year 4 but using numbers with more than 4 digits

		2	6	5	8	4		
	+	1	3	2	3	1		
		3	9	8	1	5		
				1				

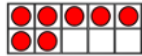
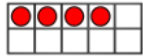
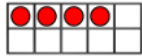
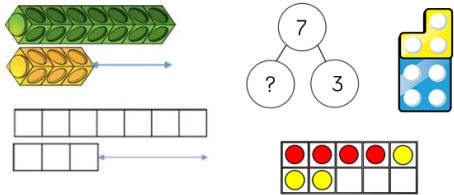

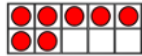
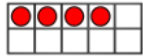
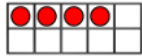
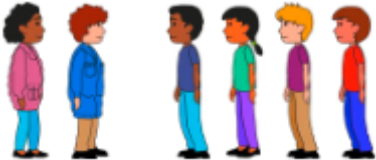

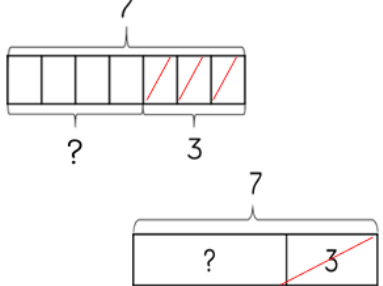
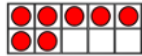
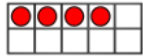
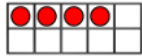
$$\begin{array}{r} 3.65 \\ + 2.41 \\ \hline 6.06 \\ \hline 1 \end{array}$$



Addition – Key mental strategies for Key Stage 1 & 2

Strategy	Concrete (Can we make it?)	Pictorial (Can we draw it?)	Abstract (Can we write the equation?)
<p>This is introduced in Year 2 but can be used up to Year 6.</p> <p><u>Bonds to 100</u></p> <p>If ___ ones + ___ ones = 10, Then ___ tens + ___ tens = 100 If I have ___ tens, I need to add ___ more tens to make 100. I need to add ___ to ___ to make 100.</p>	 <p>How many ones are there? How many tens are there? Write the number sentence for each bond. What do you notice?</p>		
<p>Bridging through a multiple of 10, 100, etc</p> <p>Years 3, 4, 5 and 6</p> <p>If ___ ones + ___ ones = 10, Then ___ tens + ___ tens = 100 If I have ___ tens, I need to add ___ more tens to make 100. I need to add ___ to ___ to make 100.</p>	 <p>7 + 5 = 7 + 3 = 10 10 + 2 = 12</p>	 <p>7 + 5 = 7 + 3 = 10 10 + 2 = 12</p>	 <p>7 + 3 = 10 10 + 2 = 12</p>
<p>Compensating – rounding to the nearest multiple 10, 100, etc and adjusting</p> <p>Years 3, 4, 5 and 6</p>	 <p>35 + 49 = 34 + 50 = 84</p>	 <p>520 + 299 = 520 + 300 = 820 820 - 1 = 819</p>	<p>69 + 69 = 138</p> <p>70 + 70 = 140</p> <p style="text-align: right;">← -2</p>

Subtraction

Stem sentences	Concrete (Can we make it?)	Pictorial (Can we draw it?)	Abstract (Can we write the equation?)						
<p>___ is the whole, ___ is a part, ___ is a part.</p> <p>___ = ___ minus ___ and ___ minus ___ = ___</p> <p>If the whole is ___ and ___ is a part, then the other part is ____.</p> <p>___ minus ___ is ____.</p> <p>___ - ____ = ____</p> <p>FS/Y1</p>	<p><i>*Part-whole models, bar models, ten frames and number shapes support partitioning.</i></p> <p><i>Ten frames, number tracks, single bar models and bead strings support reduction.</i></p> <p>Reduction – A subtraction context described by a ‘first..., then..., now...’ story is an example of this. We can link the story to a numerical representation – each number represents something in the story.</p> <div style="text-align: center; margin-top: 10px;"> <table style="margin: auto; border-collapse: collapse;"> <tr> <td style="padding: 0 10px;">First</td> <td style="padding: 0 10px;">Then</td> <td style="padding: 0 10px;">Now</td> </tr> <tr> <td style="text-align: center;"></td> <td style="text-align: center;"></td> <td style="text-align: center;"></td> </tr> </table> </div> <p><i>Cubes and bar models with two bars can support finding the difference.</i></p> <div style="text-align: center; margin-top: 10px;">  </div> <p style="margin-top: 20px;">I have 8 counters. 5 counters are red. How many are blue?</p> <div style="text-align: center; margin-top: 10px;">  </div> <p><i>In Y1, subtracting one-digit numbers that cross 10, is done by counting back, using objects, number tracks and number lines.</i></p>	First	Then	Now				<p>There are 6 children. 2 have their coat on. How many do not have their coat on?</p> <div style="text-align: center; margin-top: 10px;">  </div>	<p>There are 8 flowers. 2 are red and the rest are yellow. How many are yellow?</p> <div style="text-align: center; margin-top: 10px;">  </div> <div style="text-align: center; margin-top: 20px;">  </div>
First	Then	Now							
									

First... Then... Now...
 e.g. **First** there were 4 children in the car,
then 1 child got out. **Now** there are 3
 children in the car.

First there were ____
Then ____ were taken away.
Now there are ____.
 ____ - ____ = ____

I need to start from ____.
 I need to make ____ jumps backwards.
 I land on ____.
 This means that ____ - ____ = ____

I need to start from ____.
 I need to make ____ jumps backwards.
 I land on ____.
 This means that ____ - ____ = ____.

FS/Y1

When subtracting, the answer will be ____
 than the number I start with.

We partition the ____ into ____ and ____.
 First we subtract the ____ from ____ to get to 10.
 Then we subtract the remaining ____ from 10.
 We know 10 minus ____ is equal to ____.

I need to subtract ____ to get to 10.
 I can partition ____ into ____ and ____.
 I need to subtract ____ more.
 ____ less than ____ is ____.

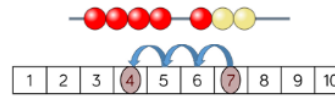
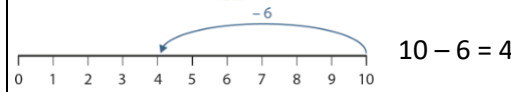
Year 2

Role play 'getting out of a car'.

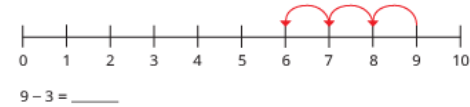
First	Then	Now
<input type="text"/>	- <input type="text"/>	<input type="text"/>
<hr/>		
<input type="text"/>	- <input type="text"/>	= <input type="text"/>



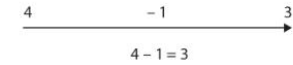
First Then Now $4 - 1 = 3$
 $3 = 4 - 1$



• Complete the number lines and the subtractions.



First Then Now



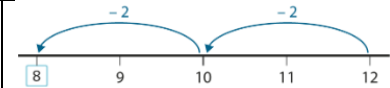
From Year 2, children should be encouraged to find the number bond to 10 when partitioning the subtracted number. Ten frames, number shapes and number lines are particularly useful for this.

$12 - 4 =$
 $12 - 2 = 10$
 $10 - 2 = 8$

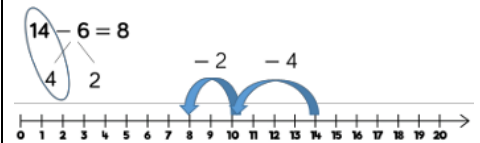
$12 - 4$

First there were 12 children on the ride.
 Then 4 got off. Now there are 8 children
 on the ride.

First Then Now

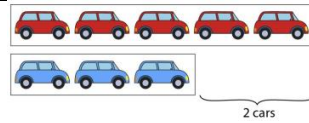


$12 - 4 =$
 $12 - 2 = 10$
 $10 - 2 = 8$

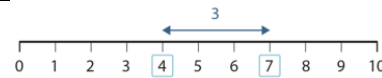


There are more ___ than ___.
 There are fewer ___ than ___.
 The difference between ___ and ___ is ___.

Year 2

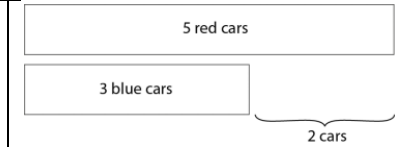


The difference between 2 and 5 is 3.
 The difference between 5 and 2 is 3.



The difference between 4 and 7 is 3.
 The difference between 7 and 4 is 3.

*Children can also use a blank number line to count back to find the difference.
 Encourage pupils to jump to multiples of 10 to become more efficient.*



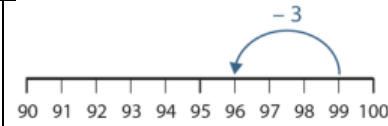
$$5 - 3 = 2$$

I know that ___ minus ___ is equal to ___.
 (single-digit fact)
 So ___ minus ___ is equal to ___. (related two-digit minus single digit fact)
 I know that ten minus ___ is equal to ___ so ___ minus ___ is equal to ___.

Year 2

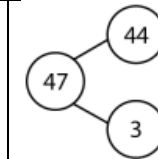
$7 - 3 = 4$
 $47 - 3 = 44$

$20 - 3 = 17$



$$9 - 3 = 6$$

$$99 - 3 = 96$$

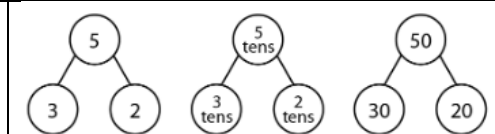
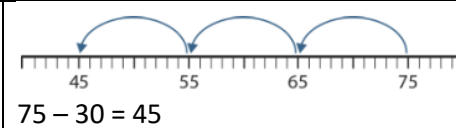


$$47 - 3 = 44$$

I know that ___ minus ___ is equal to ___.
 So ___ tens minus ___ tens is equal to ___ tens.

Year 2

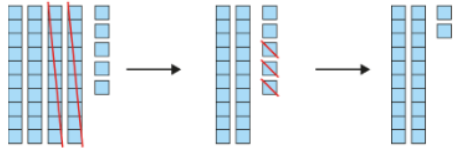
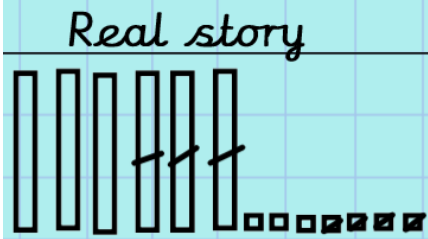
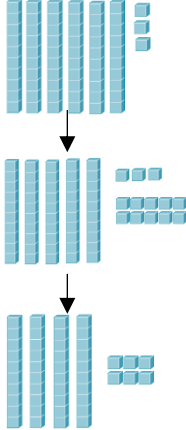
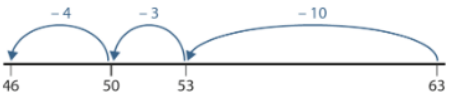
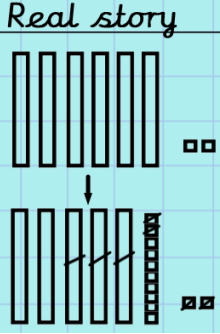
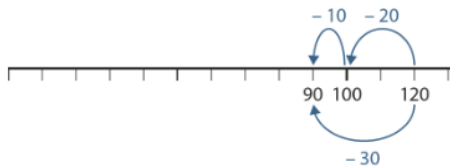
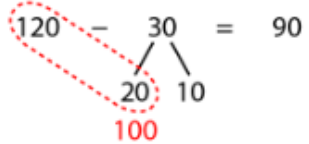
$70 - 30 = 40$ so $75 - 30 = 45$



$$5 - 3 = 2$$

$$5 \text{ tens} - 3 \text{ tens} = 2 \text{ tens}$$

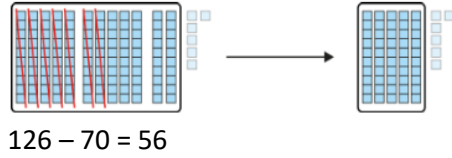
$$50 - 30 = 20$$

<p>First I subtract the tens, then I subtract the ones.</p> <p>___ ones - ___ ones = ___ ones ___ tens - ___ tens = ___ tens The difference between ___ and ___ is ___ ___ minus ___ is equal to ___</p> <p>Year 2</p>	 <p>$45 - 23 =$ $45 - 20 = 25$ $25 - 3 = 22$</p>	<p>$67 - 34 = 33$</p>  <p><i>Children can also use a blank number line to count back to find the difference. Encourage them to jump to multiples of 10 to become more efficient.</i></p>	<p>$45 - 23 = 22$</p>
<p>First I subtract the tens, then I subtract the ones.</p> <p>1 ten is equal to ___ ones. I need to exchange ___ for ____. I know I need to make an exchange because... The difference between ___ and ___ is ____.</p> <p>I know this is a subtraction because... I know I need to make an exchange because... ___ subtract ___ is equal to ____.</p> <p>Year 2</p>		 <p><i>Real story</i></p>  <p>$62 - 34 = 28$</p>	<p>$63 - 17 = 46$</p>
<p>I know that ___ minus ___ is equal to ____. (bridging ten) So ___ tens minus ___ tens is equal to ___ tens. (bridging ten tens) One hundred and ___ minus ___ is equal to ____.</p> <p>Year 3</p>	<p>See Year 2 (bridging)</p>	 <p>$120 - 30 =$ $120 - 20 = 100$ $100 - 10 = 90$</p>	 <p>$120 - 30 = 90$</p> <p>$120 - 30 =$ $120 - 20 = 100$ $100 - 10 = 90$</p>

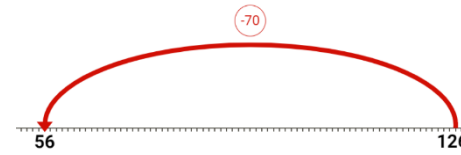
I know that ___ minus ___ is equal to ___.
 (bridging ten)
 So ___ tens minus ___ tens is equal to ___ tens.
 (bridging ten tens)
 One hundred and ___ minus ___ is equal to ___.

There are ___ hundreds, ___ tens and ___ ones.
 ___ tens minus ___ tens are equal to ___ tens.
 The tens column will decrease by ___.

Year 3



$126 - 70 = 56$



$126 - 70 = 56$

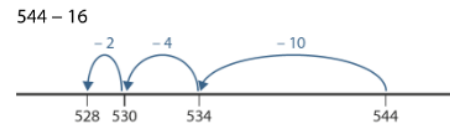
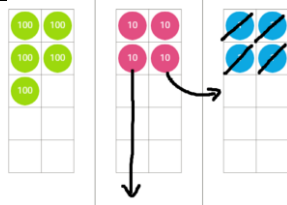
A tree diagram shows 126 branching into 120 and 6. A red dashed line connects 120 to 50 and 70, with a red circle around 50.

$126 - 70 = 120 - 70 + 6$
 $= 50 + 6$
 $= 56$

We partition the ___ into ___ and ___.
 First, we subtract the ___ from ___ to get to a multiple of 10. Then we subtract the remaining ___ from the multiple of 10. We know 10 minus ___ is equal to ___ so ___ minus ___ is equal to ___.

The previous multiple of 10 before ___ is ___.
 ___ can be partitioned into ___ and ___.
 I need to subtract ___ to get to the previous multiple of 10, then subtract ___ more.

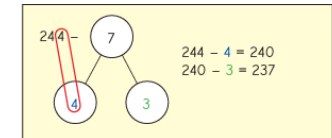
Year 3



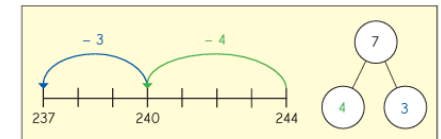
$544 - 16$

Count back to multiples of 10/100

- Scott and Whitney are working out $244 - 7$
- Scott's method**



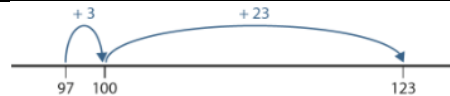
Whitney's method



We partition the ___ into ___ and ___.
 First, we add the ___ to ___ to get to 100. Then we add the remaining ___ to 100. We know 100 plus ___ is equal to ___.

Year 3

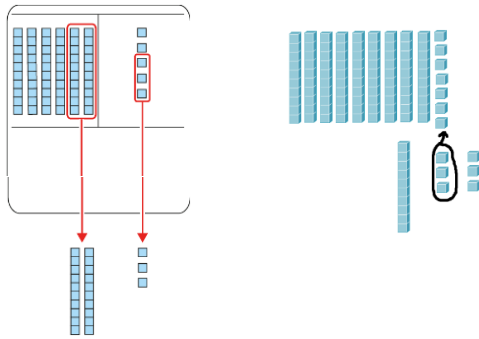
Base 10 and place value counters are the most effective manipulative when subtracting numbers with up to 3 digits.



$123 - 97 = 26$

Count on to multiples of 10/100

We line up the ones; ___ ones plus ___ ones.
 We line up the tens: ___ tens plus ___ tens.
 The ___ is in the ones column – it represents ___ ones.
 ___ ones minus ___ ones is equal to ___ ones.
 The ___ is in the tens column – it represents ___ tens.
 ___ tens minus ___ tens is equal to ___ tens.
 In column subtraction we start at the right-hand side.



$$326 - 134 =$$

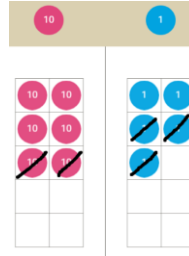
Hundreds	Tens	Ones

$$326 - 134 = 192$$

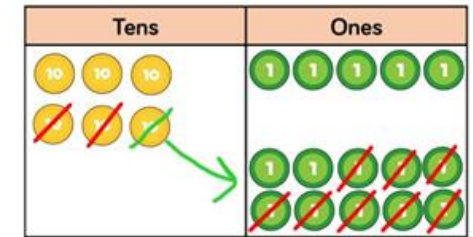
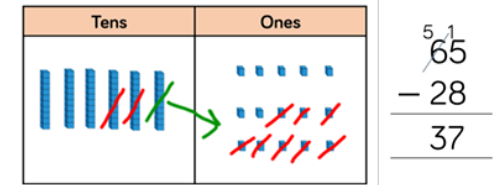
Hundreds	Tens	Ones

Year 3

Children could draw place value counters.



From Y3, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters.



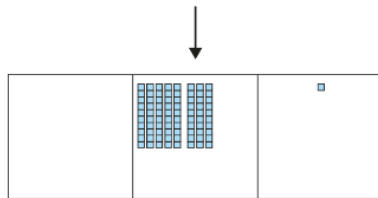
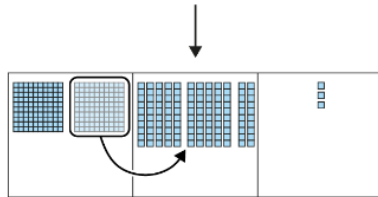
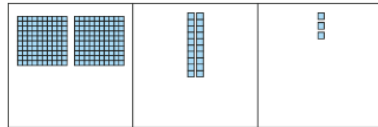
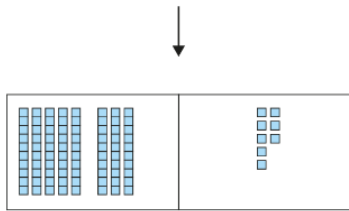
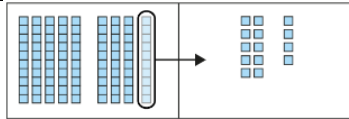
Ensure children write their calculation alongside any concrete resources so they can see the links to the written column method.
 Plain counters on a place value grid can also be used to support learning.

If there is an insufficient number to subtract from in a given column, we must exchange from the column to the left.

Year 3

___ hundreds subtract ___ hundreds is equal to ____.

I will exchange 1 hundred for ___ tens, then 1 ten for ___ ones.

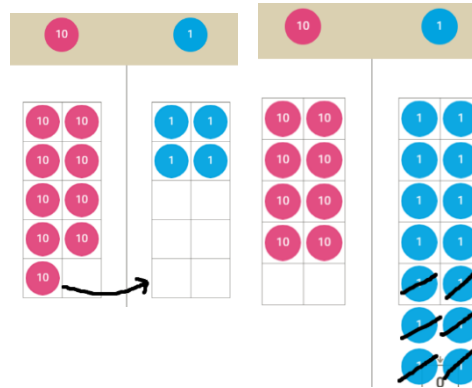


$$201 - 37 =$$

Hundreds	Tens	Ones
100 100		1

	H	T	O
	2	0	1
-		3	7

Children could draw place value counters.



Base 10 and place value counters are the most effective manipulative when subtracting numbers with up to 3 digits.

Ensure children write out their calculation alongside any concrete resources so they can see links to the written column method.

Plain counters on a PV grid can also be used to support learning.

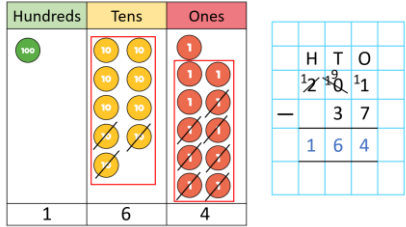
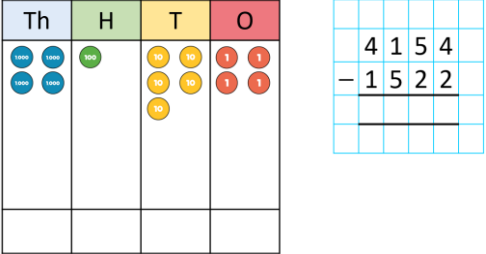
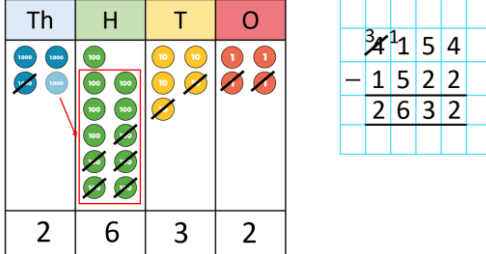
10s	1s
9 ⁸	14
-	6

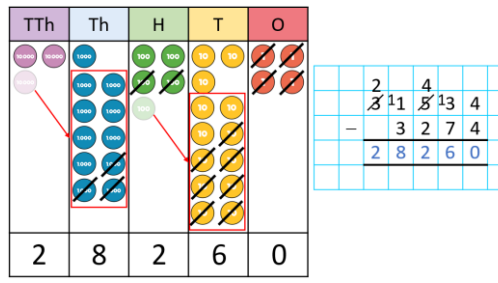
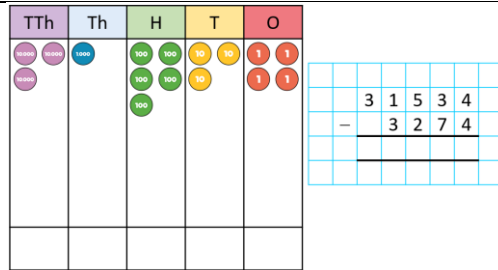
100s	10s	1s
	2	2
	4	3
	-	-

10s	1s
9 ⁸	14
-	6
8	8

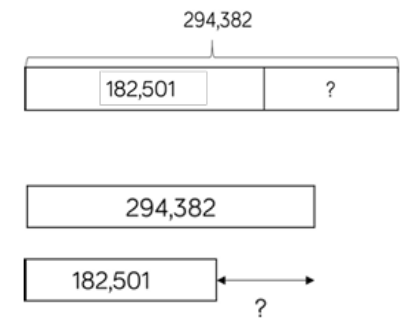
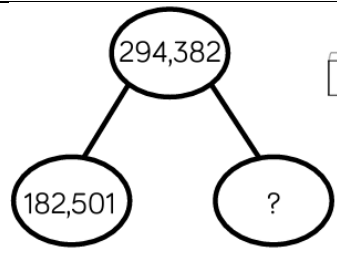
100s	10s	1s
	2 ¹	12
	4	3
	-	-

100s	10s	1s
	2 ¹	12
	4	3
	-	-
0	8	1

	<p style="text-align: center;">$201 - 37 =$</p> 		
<p>If there is an insufficient number to subtract from in a given column, we must exchange from the column to the left.</p> <p>Year 4</p>	<p>See Year 3 examples</p> <p style="text-align: center;">$4,154 - 1,522 =$</p>  <p style="text-align: center;">$4,154 - 1,522 =$</p> 	<p>See Year 3 examples</p>	<p><i>Base 10 and place value counters are the most effective manipulative when subtracting numbers with up to 4 digits.</i></p> <p><i>Ensure children write out their calculation alongside any concrete resources so they can see links to the written column method.</i></p> <p><i>Plain counters on a PV grid can also be used to support learning.</i></p> $\begin{array}{r} \overset{5}{\cancel{6}}, \overset{4}{\cancel{5}}, \overset{2}{\cancel{3}}, 18 \\ - 2, 7, 8, 9 \\ \hline 3, 7, 4, 9 \end{array}$ $\begin{array}{r} \pounds 2, \overset{8}{\cancel{9}}, \overset{14}{\cancel{5}}, 0 \\ - \pounds 1, 8, \overset{9}{\cancel{9}}, 4 \\ \hline \pounds 1, 0, \overset{5}{\cancel{5}}, 6 \end{array}$
<p>If there is an insufficient number to subtract from in a given column, we must exchange from the column to the left.</p> <p>Years 5 and 6</p>	<p>See Year 3 examples</p>	<p>See Year 3 examples</p>	<p>As in Year 4 but using numbers with more than 4 digits.</p> <p><i>At this stage, children should be encouraged to work in the abstract, using column method to subtract larger numbers efficiently.</i></p> <p><i>Ensure children have experience of subtracting decimals with a variety of decimal places. This</i></p>



Place value counters or plain counters on a place value grid are the most effective concrete resource when subtracting numbers with more than 4 digits. Counters on a place value grid are also the most effective manipulative when subtracting decimals with 1,2 and then 3 decimal places. Ensure children have experience of subtracting decimals with a variety of decimal places. This includes putting this into context when subtracting money and other measures.

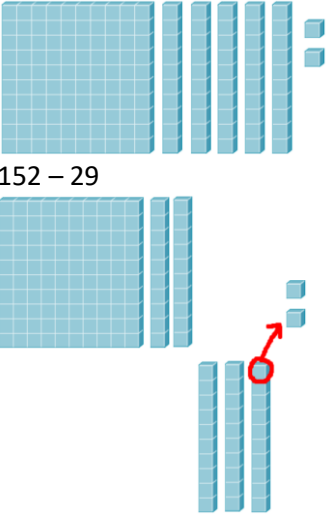
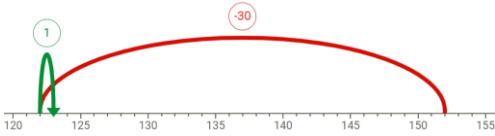


includes putting this into context when subtracting money and other measures.

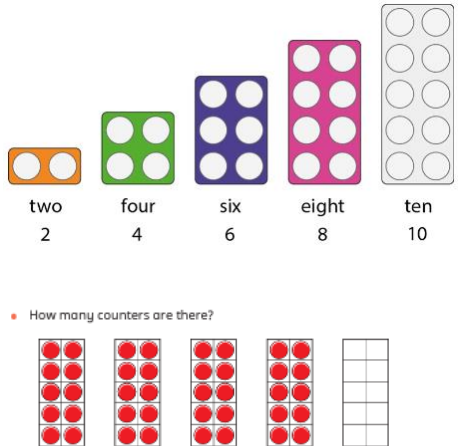
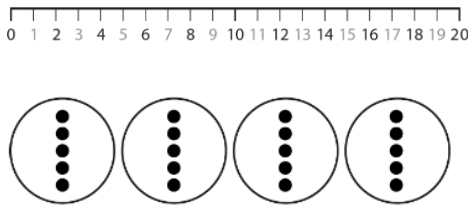
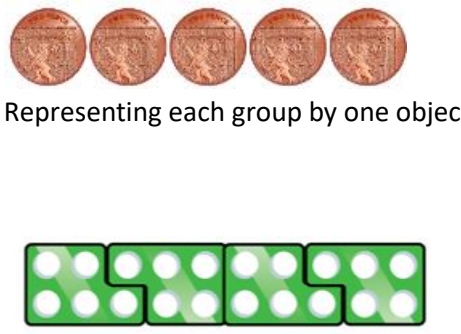
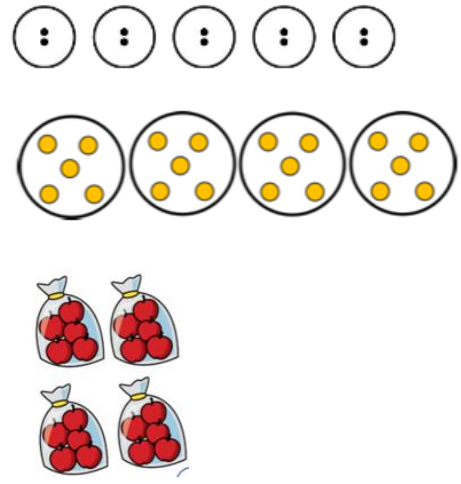
	2	9	3	1	8	2
-	1	8	2	5	0	1
<hr/>						
	1	1	1	8	8	1

Subtraction – Key mental strategies for Key Stage 2

Strategy	Concrete (Can we make it?)	Pictorial (Can we draw it?)	Abstract (Can we write the equation?)
Bridging through a multiple of 10, 100, etc Years 3, 4, 5 and 6	$12 - 4 =$ $12 - 2 = 10$ $10 - 2 = 8$	$120 - 30 =$	$120 - 30 = 90$ $120 - 30 =$ $120 - 20 = 100$

	$\begin{array}{r} 12 \\ - 4 \\ \hline 2 \quad 2 \end{array}$	$120 - 20 = 100$ $100 - 10 = 90$	$100 - 10 = 90$
<p>Compensating – rounding to the nearest multiple 10, 100, etc and adjusting</p> <p>Years 3, 4, 5 and 6</p>	 <p>$152 - 29$</p>		$152 - 30 = 122$ $122 + 1 = 123$

Multiplication

Stem sentences	Concrete (Can we make it?)	Pictorial (Can we draw it?)	Abstract (Can we write the equation?)
<p>One group of two, two groups of two, three groups of 2, ...</p> <p>Ten, twenty, thirty, ...</p> <p>One five, two fives, three fives, ...</p> <p>There are ___ groups of ten. There are ___ altogether.</p> <p>There are ___ full ten frames. There are ___ in total.</p> <p>FS/Y1</p>	 <p>two four six eight ten 2 4 6 8 10</p> <p>• How many counters are there?</p>		<p>10, 20, 30, ...</p>
<p>There are ___ coins. Each coin has a value of ___p. This is ___p.</p> <p>There are ___ equal groups of ____. I know that the groups are equal/not equal because... To make the groups equal, I could...</p> <p>There are ___ equal groups of ____. I know that the groups are equal/not equal because... To make the groups equal, I could...</p> <p>There are ___ equal groups. There are ___ in each group. There are ___ altogether. There are ___ groups of ____ ___ + ___ + ___ + ___ + ___ = ___</p> <p>Year 1</p>	 <p>Representing each group by one object</p> <p><i>Children represent multiplication as repeated addition in many different ways. In Y1, children use concrete and pictorial representations to solve problems. They are not expected to record multiplication formally.</i></p>		<p>Five 2p coins = 10p</p> <p>$5 + 5 + 5 + 5 = 20$</p> <p>$5 + 5 + 5 + 5 = 20$</p>

There are ___ in each group.
 There are ___ groups.
 There are ___ in a group and ___ groups.

Year 2

There are ___ equal groups with ___ in each group.
 There are ___ groups of ___.
 There are ___ altogether.

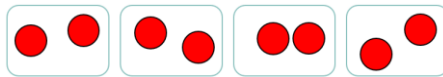
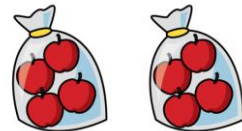
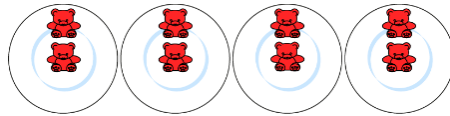
___ lots of ___ = ___
 ___ multiplied by ___ is equal to ___
 ___ x ___ = ___

___ x 2 is the same as ___ lots of 2.
 ___ multiplied by 2 is equal to ___.
 I know that ___ x 2 = ___, so I can add/subtract 2 to work out ___ x 2.

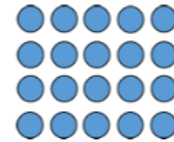
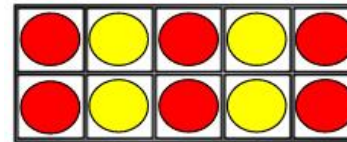
In Year 2, children are introduced to the multiplication symbol.

Encourage daily counting in multiples both forwards and backwards. This can be supported using a number line or a hundred square.

Look for patterns in the two times table, using concrete manipulatives to support. Notice how all the numbers are even and there is a pattern in the ones.



5	5	5
---	---	---



$$5 + 5 + 5 + 5 = 20$$

$$4 \times 5 = 20$$

$$5 \times 4 = 20$$

$$2 + 2 + 2 + 2 = 8$$

$$4 \times 2 = 8$$

$$5 + 5 + 5 = 15$$

$$3 \times 5 = 15$$

3 lots of 5 equal 15.

6 lots of 2 equal 12.

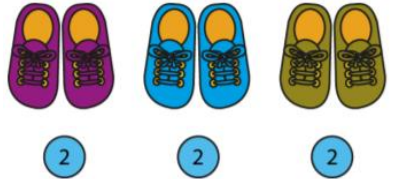
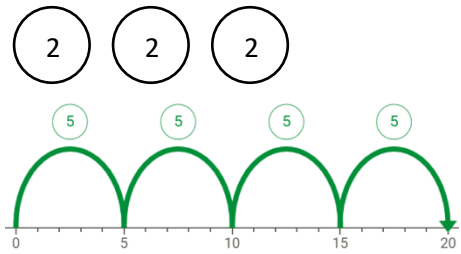
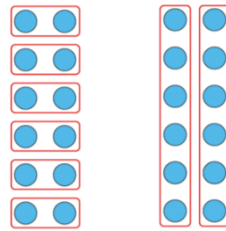
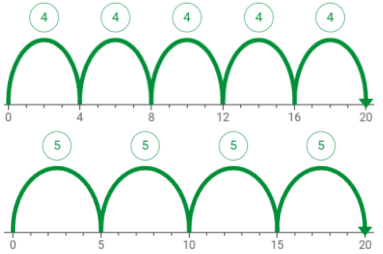
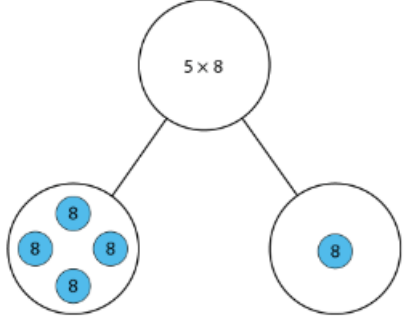
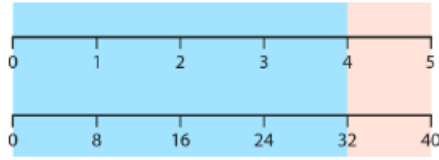
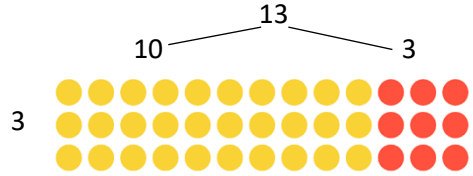
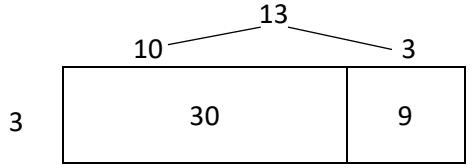
In Y2, children are introduced to the multiplication symbol. Children also make the link between multiplication and repeated addition. Children should already be secure in identifying equal groups and be able to represent this as an addition number sentence.

When writing factor times factor, the first factor is the number of groups x the number within each group.

$$3 \times 4$$

There are 3 groups/lots of 4.

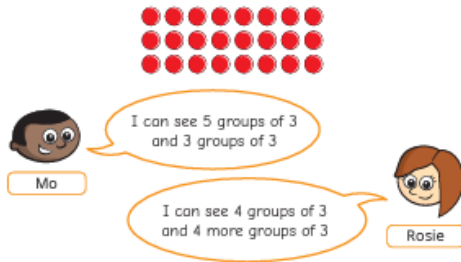
Children may find that using the language 'lots of' builds on previous learning, but they should also use other variations interchangeably, such as "times," "multiplied by" and so on.

<p>Factor times factor is equal to the product. The product is equal to factor times factor.</p> <p>There are 3 equal groups with ___ in each group. ___ + ___ + ___ + ___ = ___ ___ x ___ = ___</p> <p>Year 2</p>	 <p>Unitising equal groups – representing each group by one object</p>		$3 \times 2 = 6$ $6 = 3 \times 2$
<p>___ times ___ can represent ___ in a group and ___ groups. It can also represent ___ groups of ___.</p> <p>Multiplication is commutative.</p> <p>Year 2</p>			$2 \times 5 = 5 \times 2$
<p>___ is equal to ___ plus ___, so ___ times ___ is equal to ___ times ___ plus ___ times ___.</p> <p>___ is equal to ___ minus ___, so ___ times ___ is equal to ___ times ___ minus ___ times ___.</p> <p>Multiplication is distributive.</p> <p>(NCETM Year 4 unit 2.10)</p> <p>Year 3</p>			$5 = 4 + 1$ $5 \times 8 = 4 \times 8 + 1 \times 8$ $= 32 + 8$ $= 40$ $4 = 5 - 1$ $4 \times 8 = 5 \times 8 - 1 \times 8$ $= 40 - 8$ $= 32$
<p>___ is equal to ___ plus ___, so ___ times ___ is equal to ___ times ___ plus ___ times ___.</p> <p>___ is equal to ___ minus ___, so ___ times ___ is equal to ___ times ___ minus ___ times ___.</p>			$3 \times 13 = 3 \times 10 + 3 \times 3$ $= 30 + 9$ $= 39$

Multiplication is distributive.

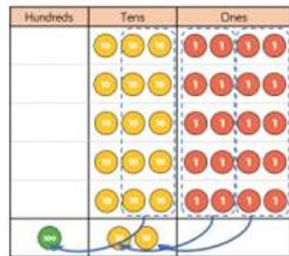
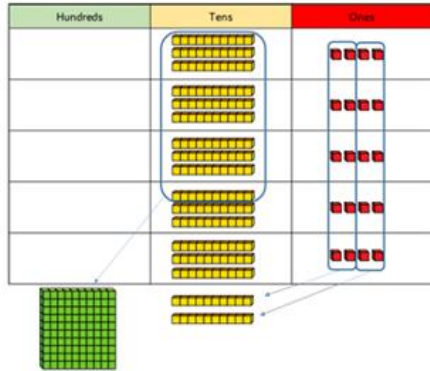
There are ___ groups.
 There are ___ in each group.
 There are ___ altogether.
 $__ \times 3 = __ \times 3 + __ \times 3$

(NCETM Year 4 unit 2.10)
Year 3



Informal methods and the expanded method are used in Year 3 before moving on to the short multiplication method in Year 4. Children apply their understanding of partitioning to represent and solve calculations using the expanded method. This involves partitioning the 2-digit number into tens and ones, multiplying separately, then adding the partial products together.

___ tens and ___ ones multiplied by ___ is equal to ___ tens multiplied by ___ and ___ ones multiplied by ___.
 ___ ones is ___ tens and ___ ones.
 $__ \times __ = __ \text{ tens} \times __ + __ \times __$



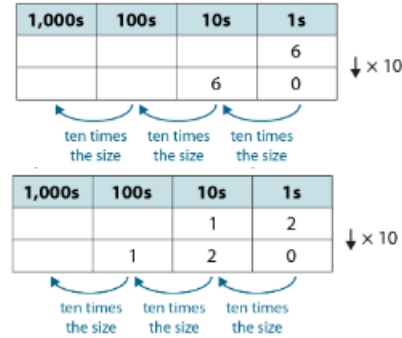
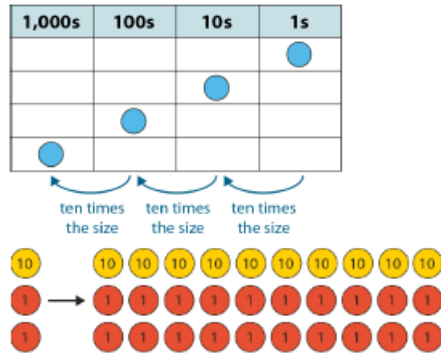
	H	T	O		
		3	4		
x			5		
		2	0	(5 × 4)	
+	1	5	0	(5 × 30)	
	1	7	0		

Year 3

To multiply a whole number by 10, place a zero after the final digit of that number.

Year 4

___ x 10 = ___
 10 x ___ = ___
 ___ is 10 times the size of ___



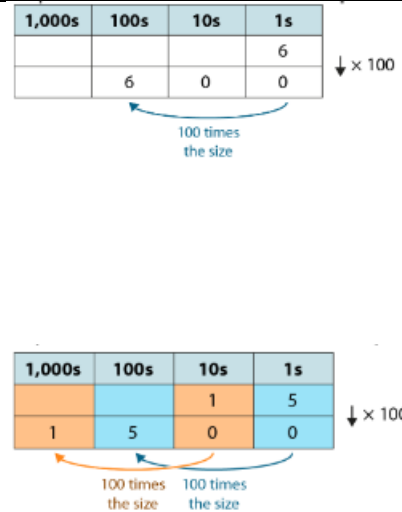
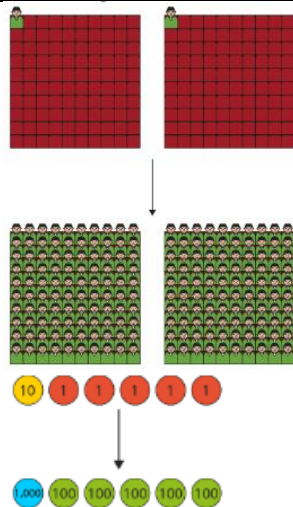
$6 \times 10 = 60$

$12 \times 10 = 120$

All multiples of 100 have both a tens and ones digit of 0.
 When a number is multiplied by 100, the product is a multiple of 100.

Year 4

___ x 100 = ___ x 10 x 10 = ___ x 10 = ___
 ___ x 100 = ___, so 100 x ___ = ___
 ___ is 100 times the size of ___.



$2 \times 100 = 200$

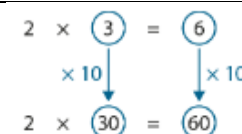
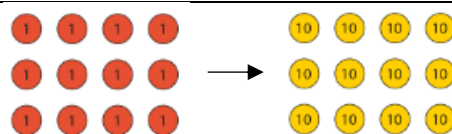
There are 100 times as many people as before.

$15 \times 100 = 1500$

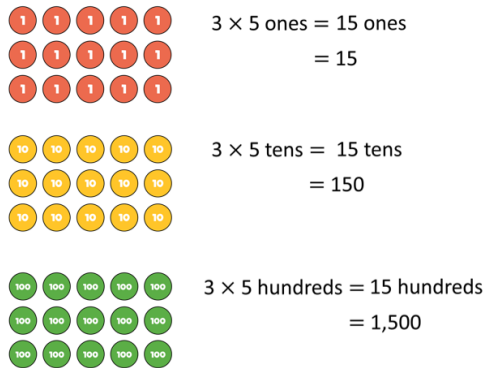
If one factor is made ten times the size, the product will be ten times the size.

Year 4

___ x 10 = ___
 10 x ___ = ___
 ___ is 10 times the size of ___



$4 \times 3 = 12$ so $4 \times 30 = 120$



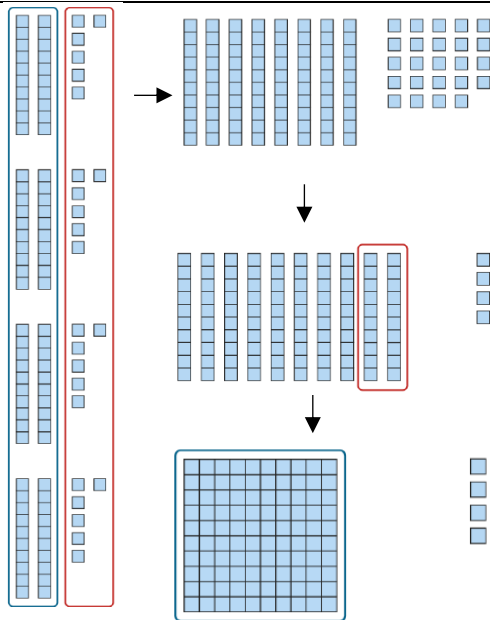
If there are ten or more ones, we must exchange the ones into tens and ones. If there are ten or more tens, we must exchange the tens into hundreds and tens.

Multiplication is distributive.

Year 4

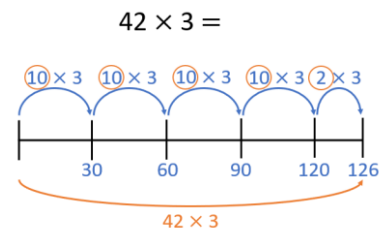
$_ \times _ = _ \times _ + _ \times _$
 $_ \times _ = _ \times _ - _ \times _$
 $_ \times _ = _ \times _ \times 2$
 $_ \times _ = _ \times _ \div 2$

In this step, children consolidate their knowledge and understanding of multiplication and begin to make decisions regarding the most efficient or appropriate methods to use in a range of contexts.



$84 \times 6 = 504$

$80 \times 6 = 480$
 $4 \times 6 = 24$
 $480 + 24 = 504$



$84 \times 6 = 80 \times 6 + 4 \times 6$
 $= 480 + 24$
 $= 504$

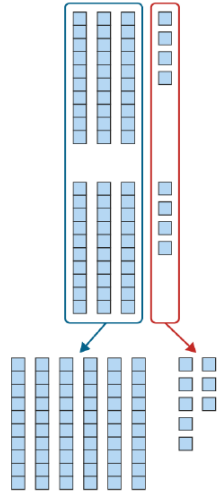
$24 \times 4 = 20 \times 4 + 4 \times 4$
 $= 80 + 16$
 $= 96$

Informal methods and the expanded method are used in Y3 before moving on to the short multiplication method in Y4. Place value counters should be used to support the understanding of the method rather than supporting the multiplication, as children should use times table knowledge.

We work from the least significant digit, on the right, to the most significant digit, on the left.

Multiplication is distributive.

Year 4



$$34 \times 2 = 60 + 8 = 68$$

	10s	1s
3	4	
×		2
		8
	6	0
	6	8
		↓
	2	1
×		4
	8	4

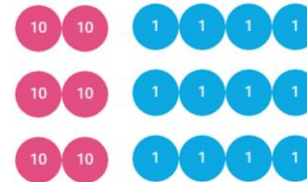
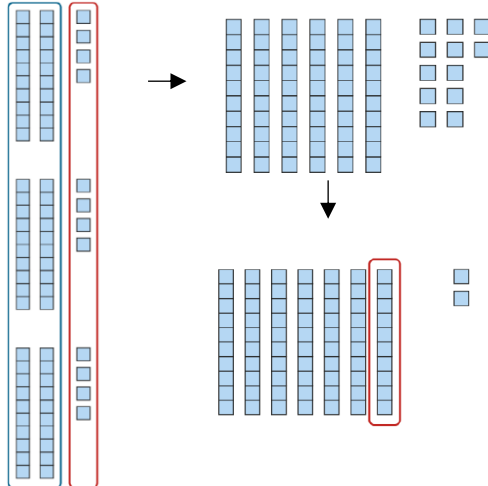
$2 \times 4 \text{ ones} = 8 \text{ ones}$

$2 \times 3 \text{ tens} = 6 \text{ tens}$

If there are ten or more ones, we must exchange the ones into tens and ones. If there are ten or more tens, we must exchange the tens into hundreds and tens.

Multiplication is distributive.

Year 4



$$24 \times 3 = 60 + 12 = 72$$

	10s	1s
2	4	
×		3
	1	2
	6	0
	7	2
		↓
	1	8
×		5
	9	0
	4	

$3 \times 4 \text{ ones} = 12 \text{ ones} = 1 \text{ ten} + 2 \text{ ones}$

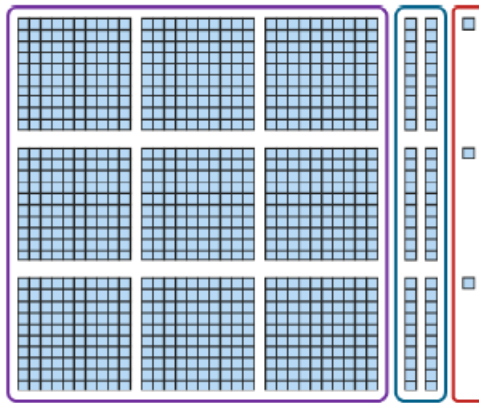
$3 \times 2 \text{ tens} = 6 \text{ tens}$

		3	8
		1	5
		1	5
		3	2
			3

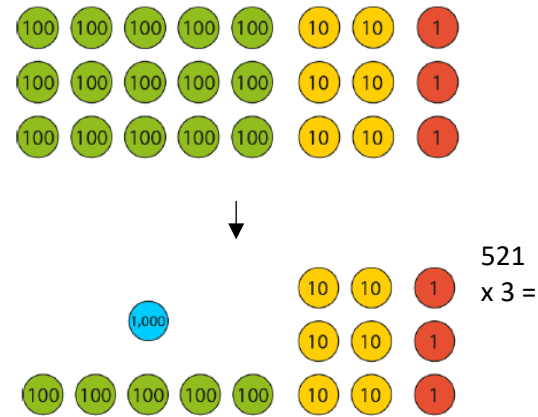
If there are ten or more ones, we must exchange the ones into tens and ones.
 If there are ten or more tens, we must exchange the tens into hundreds and tens.
 If there are ten or more hundreds, we must exchange the hundreds into thousands and hundred.

Multiplication is distributive.

Year 4



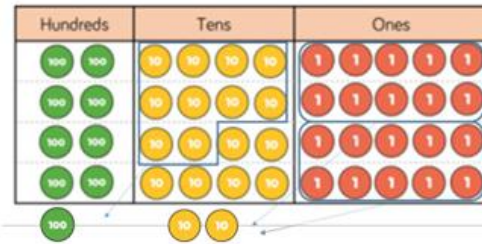
$321 \times 3 = 963$



$1000 + 500 + 60 + 3 = 1563$

Hundreds	Tens	Ones
2	4	5
x		
4		
9 8 0		
1 2		

$245 \times 4 = 980$



100s	10s	1s
3	2	1
x		
3		
9 6 3		
3 2 1		
x		
3		
9 6 3		

$3 \times 1 \text{ ones} = 3 \text{ ones}$
 $3 \times 2 \text{ tens} = 6 \text{ tens}$
 $3 \times 3 \text{ hundreds} = 9 \text{ hundreds}$

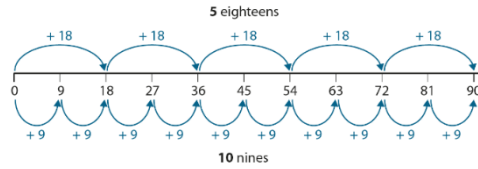
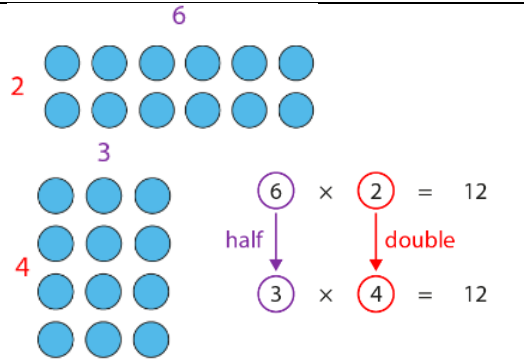
1,000s	100s	10s	1s
	5	2	1
x			
3			
15 6 3			
3 6 7			
x			
4			
1 4 6 8			
2 2			

When moving to 3-digit by 1-digit multiplication, encourage children to move towards the short, formal written method. Base 10 and place value counters continue to support the understanding of the written method. Limit the number of exchanges needed in the questions and move children away from resources when multiplying larger numbers.

If there is a multiplicative increase in one factor and a multiplicative decrease in the other, the product remains the same.

If I multiply one factor by __, I must divide the other factor by __ for the product to remain the same.

Year 5 and 6



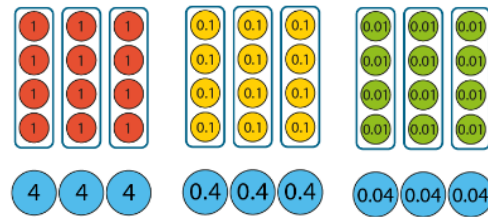
$$\begin{array}{c} 2 \\ \times 3 \\ \hline 6 \end{array} \times \begin{array}{c} 9 \\ \div 3 \\ \hline 3 \end{array} = 18$$

If one factor is made one tenth of the size, the product will be one tenth of the size.

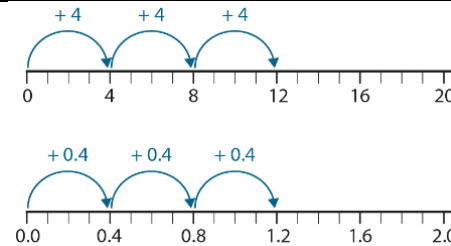
If one factor is made one hundredth of the size, the product will be one hundredth of the size.

I move the digits of the number I am multiplying __ places to the left until I get a whole number; then I multiply; then I move the digits of the product __ places to the right.

Year 5



$$\begin{array}{l} 4 \times 3 = 12 \\ 0.4 \times 3 = 1.2 \\ 0.04 \times 3 = 0.12 \end{array}$$



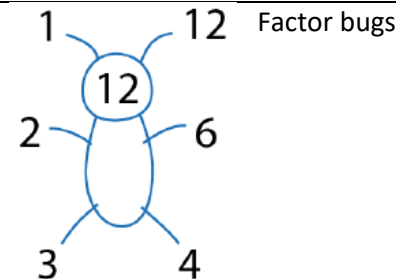
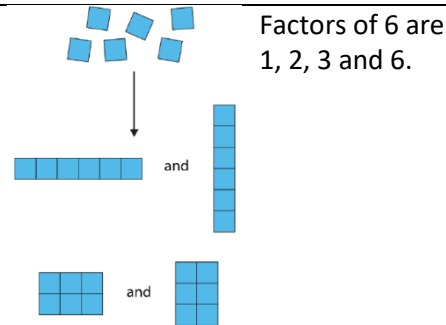
$$\begin{array}{r} 4 \ 5 \ 6 \\ \times \quad \quad 4 \\ \hline 1 \ 8 \ 2 \ 4 \\ \hline 2 \ 2 \end{array}$$

$$\begin{array}{r} 4 \ . \ 5 \ 6 \\ \times \quad \quad 4 \\ \hline 1 \ 8 \ . \ 2 \ 4 \\ \hline 2 \ 2 \end{array}$$

Numbers that have more than two factors are composite numbers.

Year 5

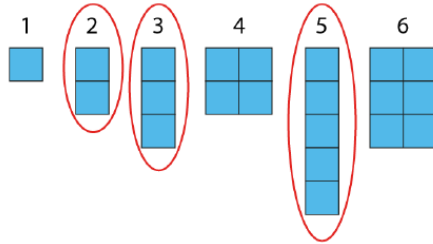
The only factors of __ are __ and __, so __ is prime.
__ is prime and a factor of __, so __ is a prime factor of __.



Factors of 6 are 1, 2, 3 and 6.

Numbers that have only two factors are prime numbers.

Year 5



17 is a prime number because its only factors are 1 and 17.

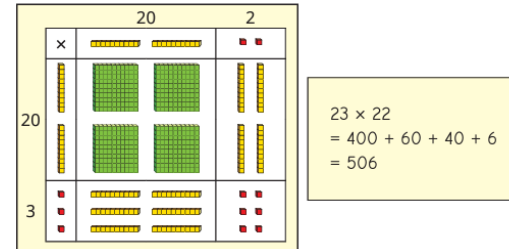
Year 5

When multiplying a multi-digit number by 2-digits, use the area model to help children understand the size of the numbers they are using.

This links to finding the area of a rectangle by finding the space covered by the Base 10. The grid method matches the area model as an initial written method before moving on to the formal written multiplication method.

___ ones x ___ = ___ ones, so ___ tens x ___ = ___ tens

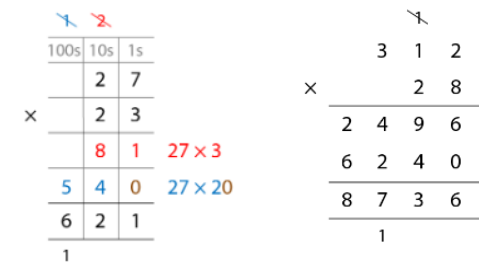
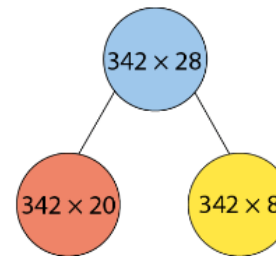
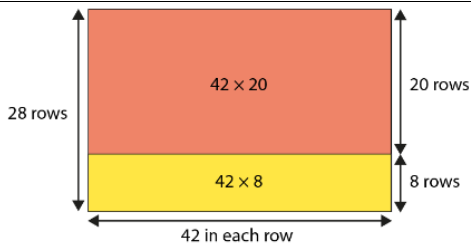
The products in my area model are ___, ___, ___ and ___, so the total product is ___ + ___ + ___ + ___ = ___



×	20	2
30	600	60
1	20	2

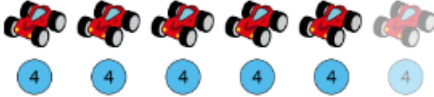
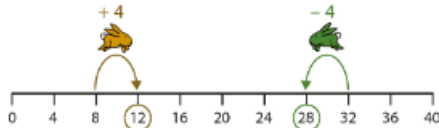
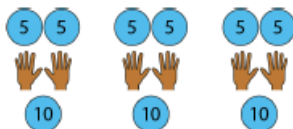
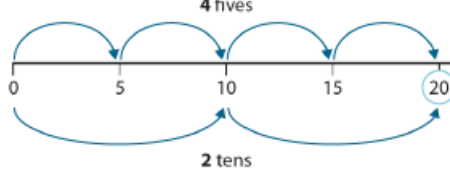
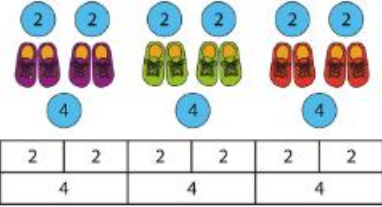
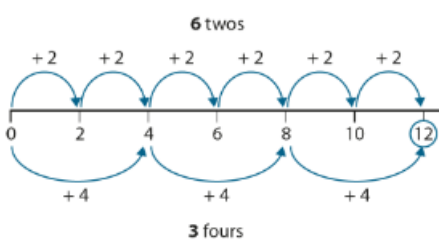
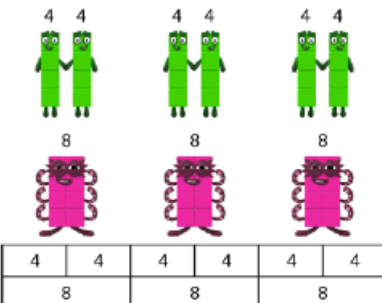
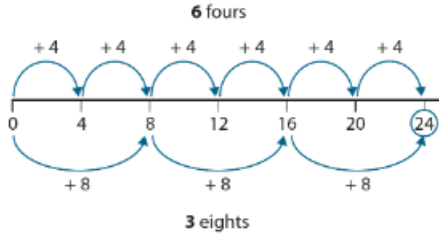
To multiply two two-digit numbers, first multiply by the ones, then multiply by the tens, then add them together.

To multiply a three-digit number by a two-digit number, first multiply by the ones, then multiply by the tens, then add them together.



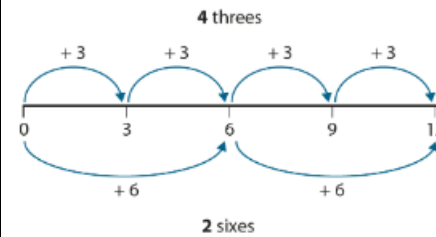
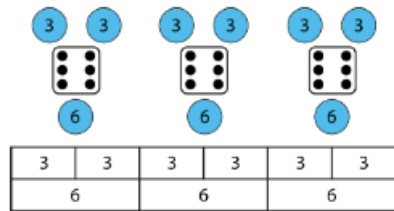
Year 6

Multiplication – Key mental strategies for Key Stage 2

Strategy	Concrete (Can we make it?)	Pictorial (Can we draw it?)	Abstract (Can we write the equation?)
<p>Adjacent multiples of ___ have a difference of ___.</p> <p>Year 3 onwards</p>			<p>$4 \times 6 = 4 \times 5 + 4$</p> <p>$4 \times 9 = 4 \times 10 - 4$</p>
<p>Products in the 10 times table are double the products in the 5 times table. Products in the 5 times table are half of the products in the 10 times table.</p> <p>(NCETM Year 2 unit 2.5)</p> <p>Year 3 onwards</p>			<p>$5 \times 4 = 10 \times 2$</p>
<p>Products in the 4 times table are double the products in the 2 times table. Products in the 2 times table are half of the products in the 4 times table.</p> <p>Year 3 onwards</p>			<p>$2 \times 6 = 4 \times 3$</p>
<p>Products in the 8 times table are double the products in the 4 times table. Products in the 4 times table are half of the products in the 8 times table.</p> <p>Year 3 onwards</p>			<p>$4 \times 6 = 8 \times 3$</p>

Products in the 6 times table are double the products in the 3 times table.
 Products in the 3 times table are half of the products in the 6 times table.

Year 3 onwards

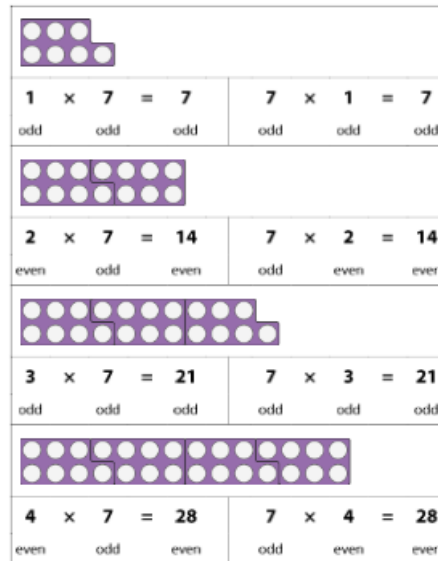


$3 \times 4 = 6 \times 2$

When both factors are odd, the product is odd.
 When one factor is odd and the other factor is even, the product is even.

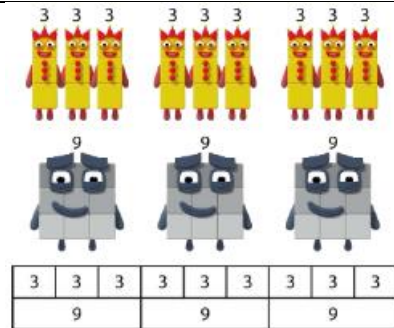
(NCETM Year 3 unit 2.9)

Year 3 onwards



odd x odd = odd
 odd x even = even
 even x odd = even
 even x even = even

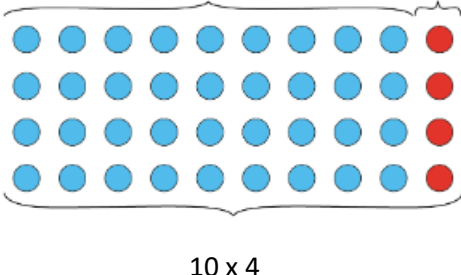
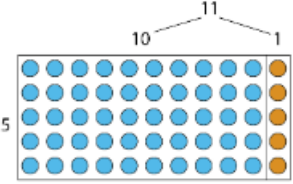
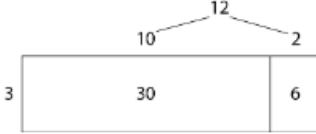
Products in the 9 times table are triple the products in the 3 times table.




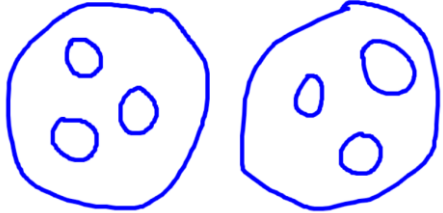

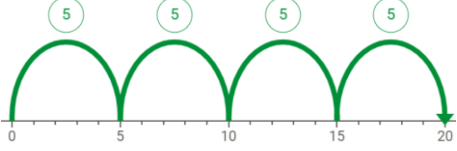
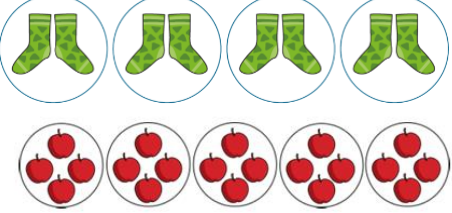
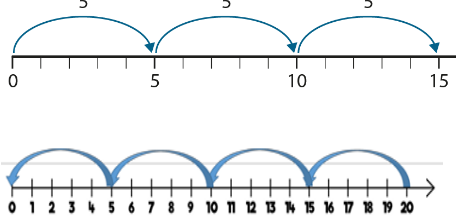
9×4 1×4

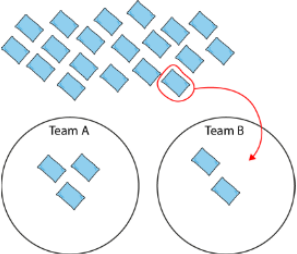
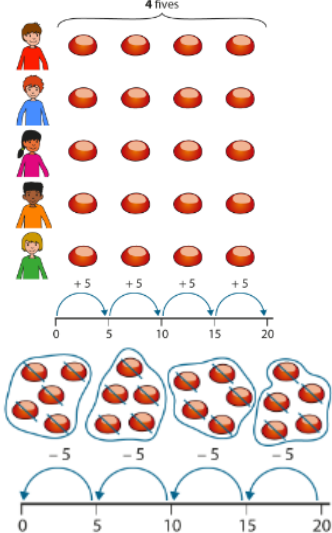
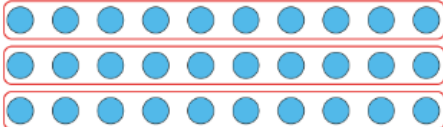
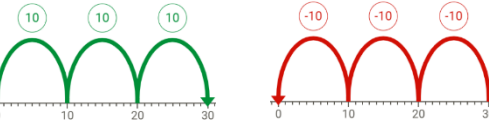
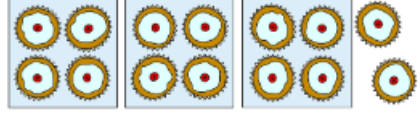
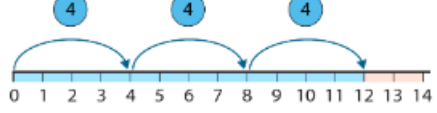
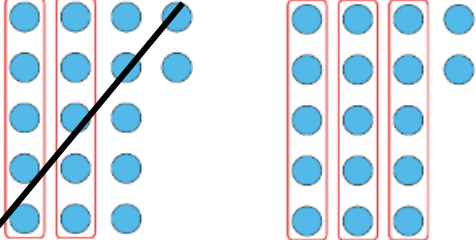
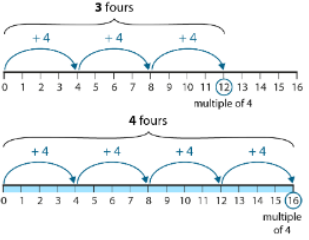


$3 \times 12 = 9 \times 4$

<p>Products in the 10 times table can be used to find products in the 9 times table.</p> <p>(NCETM Year 3 unit 2.8)</p> <p>Year 4 onwards</p>	 <p style="text-align: center;">10×4</p>		$9 \times 4 = 10 \times 4 - 1 \times 4$
<p>Products in the 10 times table can be used to find products in the 11 times table and 12 times table.</p> <p>Year 4 onwards</p>	 <p style="text-align: center;">10×4</p>		$\begin{aligned} 12 \times 3 &= 10 \times 3 + 2 \times 3 \\ &= 30 + 6 \\ &= 36 \end{aligned}$

Division

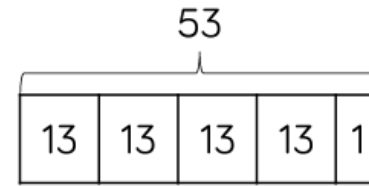
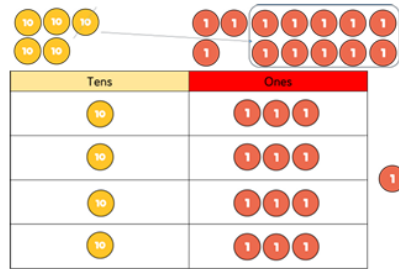
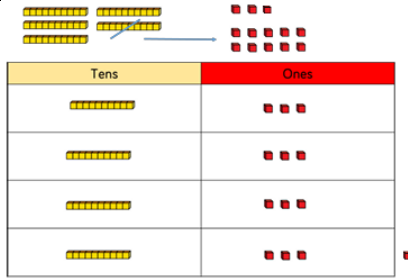
Stem sentences	Concrete (Can we make it?)	Pictorial (Can we draw it?)	Abstract (Can we write the equation?)
<p>One group of two, two groups of two, three groups of 2, ...</p> <p>Ten, twenty, thirty, ...</p> <p>One five, two fives, three fives, ...</p> <p>FS/Y1</p>			<p>6 biscuits shared between 2 children gives 3 biscuits each.</p>
<p>The ____ costs __p.</p> <p>Each coin has a value of __p.</p> <p>So I need __ coins.</p> <p>Year 1</p>			<p>Five 2p coins = 10p</p>
<p>__ is divided into groups of __.</p> <p>There are __ groups.</p> <p>We can skip count using the divisor to find the quotient.</p> <p><i>Children use concrete and pictorial representations to solve problems. They are not expected to record division formally. Children solve problems by grouping and counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction on a number line.</i></p> <p>Year 2</p>			<p>$5 + 5 + 5 = 15$</p> <p>$15 \div 5 = 3$</p> <p><i>In Year 2, the children are introduced to the division symbol.</i></p>

<p>___ divided between ___ is equal to ___ each.</p> <p>We can skip count using the divisor to find the quotient.</p> <p><i>Children solve problems by sharing amounts into equal groups.</i></p> <p>Year 2</p>			<p>One 5 is 1 each. That's 5. Two 5s is 2 each. That's 10. $10 \div 5 = 2$</p>
<p>Ten times ___ is equal to ___ so ___ divided into groups of ten is ___.</p> <p>If the divisor is ___, we can use the ___ times table to find the quotient.</p> <p>Year 2</p>	 <p>30 represents the total number of counters. 10 represents the number in each group. 3 represents the number of groups.</p>		<p>$10 \times 3 = 30$ $3 \times 10 = 30$ $30 \div 10 = 3$</p>
<p>___ is divided into groups of ___. There are ___ groups and a remainder of ___.</p> <p>(NCETM Year 4 unit 2.12)</p> <p>Year 3</p>			<p>$14 = 4 \times 3 + 2$ $14 \div 4 = 3 \text{ r } 2$</p>
<p>___ is a multiple of ___ so when it is divided into groups of ___, there is no remainder.</p> <p>The remainder is always less than the divisor.</p>			<p>$17 \div 5 = 2 \text{ r } 7$ is incorrect because 7 is greater than 5. $17 \div 5 = 3 \text{ r } 2$</p>

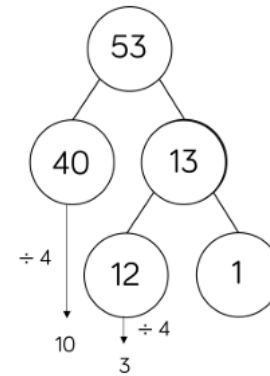
When dividing numbers with remainders, children can use Base 10 and place value counters to exchange one ten for ten ones. Starting with equipment outside the place value grid will highlight remainders, as they will be left outside the grid once the equal groups have been made. Flexible partitioning in a part-whole model supports this method.

There are ___ groups of ___
 There are ___ remaining.
 So ___ ÷ ___ = ___ r ___

(NCETM Year 4 unit 2.12)
Year 3 or 4?



$$53 \div 13 = 4 \text{ r } 1$$

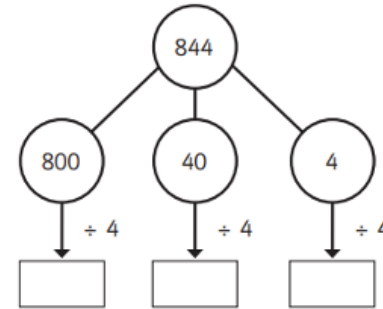
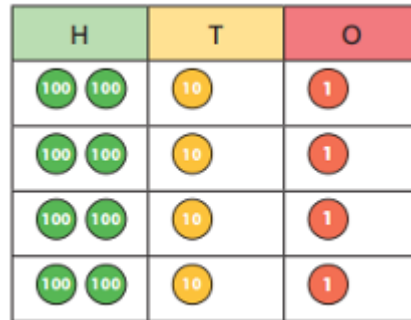


Divide 3-digits by 1-digit (sharing)

Children can continue to use place value counters to share 3-digit numbers into equal groups. Children should start with the equipment outside the place value grid before sharing the hundreds, tens and ones equally between rows. This method might help highlight remainders. Flexible partitioning in a part-whole model supports this method.

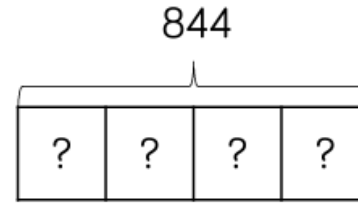
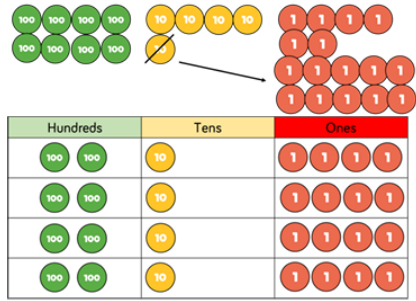
___ hundreds divided by ___ = ___ hundreds
 ___ tens divided by ___ = ___ tens
 ___ ones divided by ___ = ___ ones
 There is ___ left over, so I need to exchange it for ___

Year 4



$$646 \div 2 = 300 + 20 + 3 = 323$$

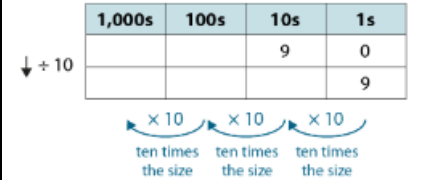
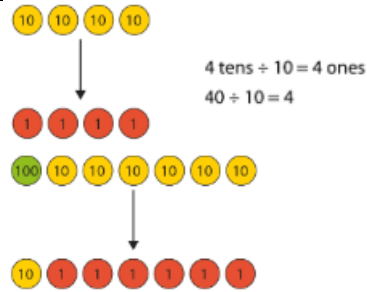
$$844 \div 4 = 200 + 10 + 1 = 211$$



To divide a multiple of ten by 10, remove the zero from the ones place.

$\underline{\quad} \div 10 = \underline{\quad}$
 $\underline{\quad} = \underline{\quad} \div 10$
 $\underline{\quad}$ is one-tenth the size of $\underline{\quad}$

Year 4



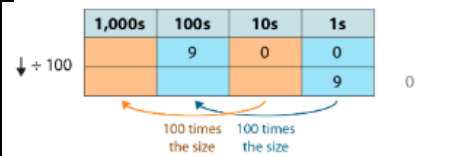
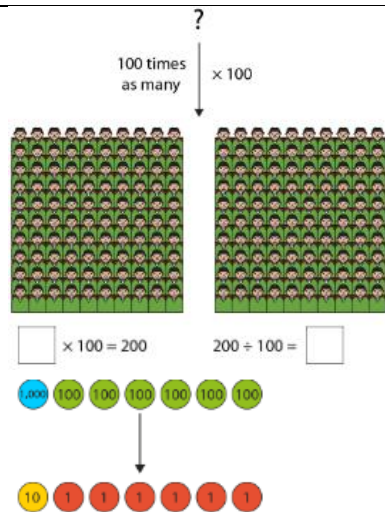
$90 \div 10 = 9$

$150 \div 10 = 15$

To divide a multiple of 100 by 100, remove two zeros (from the tens and ones places).

$\underline{\quad} \div 100 = \underline{\quad} \div 10 \div 10 = \underline{\quad} \div 10 = \underline{\quad}$
 $\underline{\quad} \div 100 = \underline{\quad}$, so $\underline{\quad} = \underline{\quad} \div 100$
 $\underline{\quad}$ is one-hundredth the size of $\underline{\quad}$

Year 4

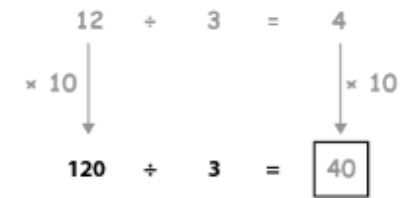
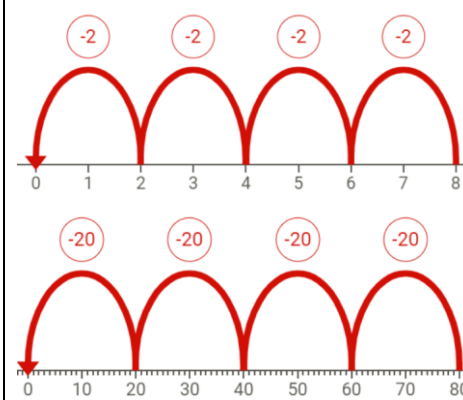
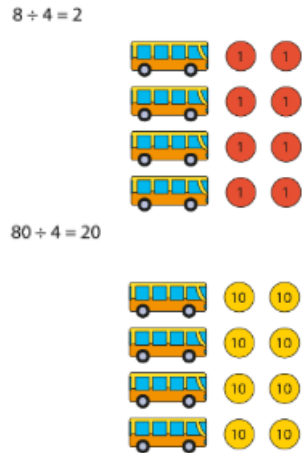


$900 \div 100 = 9$

$1500 \div 100 = 15$

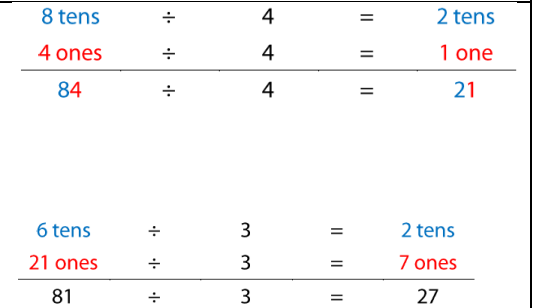
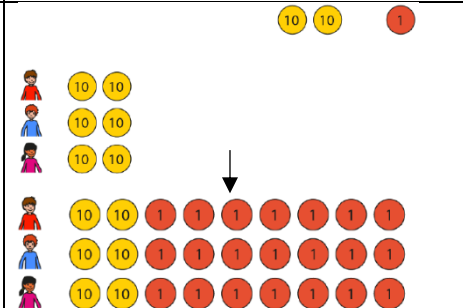
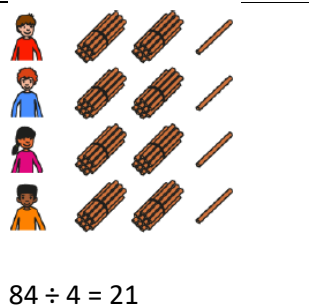
If the dividend is made ten times the size, the quotient will be ten times the size.

Year 4



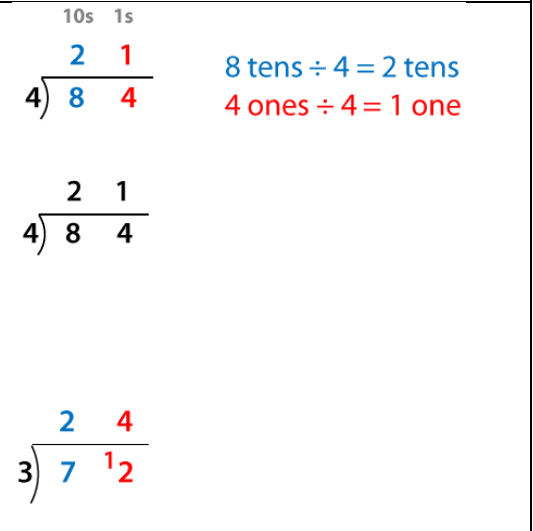
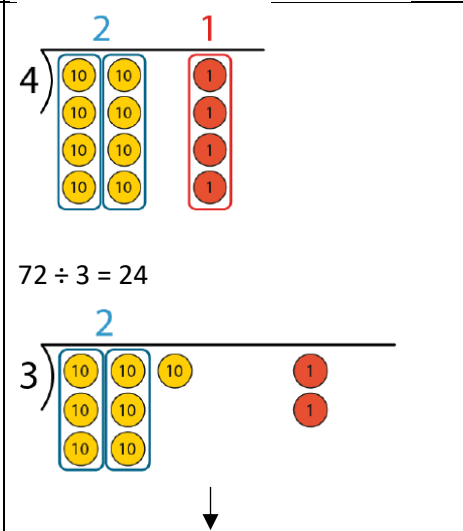
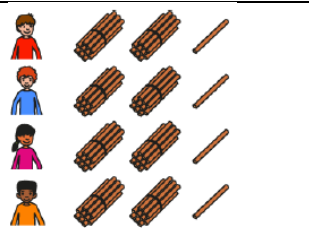
If dividing the tens gives a remainder of one or more tens, we must exchange the remaining tens for ones.

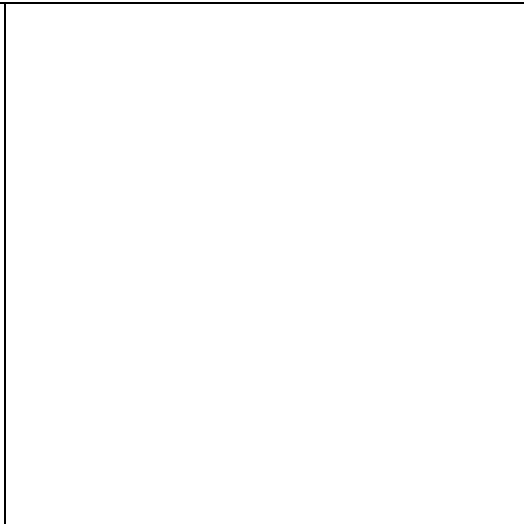
Year 4



If dividing the tens gives a remainder of one or more tens, we must exchange the remaining tens for ones.

Year 4





2 4

$$3 \overline{) \begin{array}{cc} 10 & 10 \\ 10 & 10 \\ 10 & 10 \end{array} \begin{array}{cccc} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{array}}$$

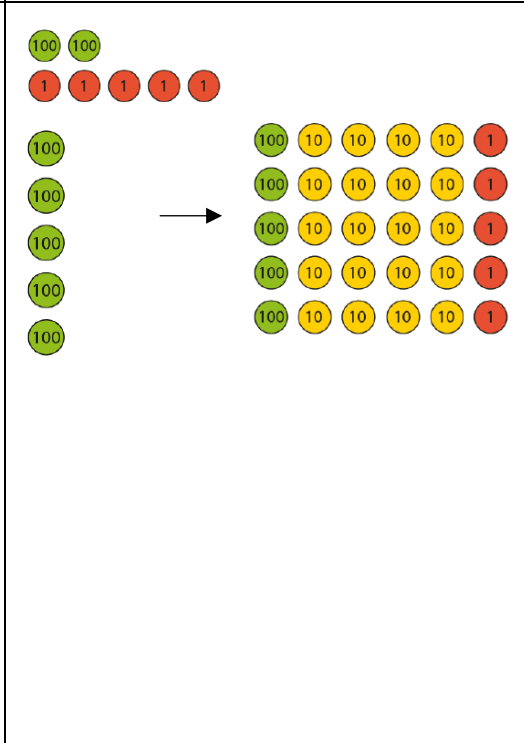
$73 \div 3 = 24 \text{ r } 1$

2 4 r1

$$3 \overline{) \begin{array}{cc} 10 & 10 \\ 10 & 10 \\ 10 & 10 \end{array} \begin{array}{cccc} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{array} 1$$
$$3 \overline{) \begin{array}{r} 24 \text{ r } 1 \\ 73 \end{array}}$$

If dividing the hundreds gives a remainder of one or more hundreds, we must exchange the remaining hundreds for tens.

Year 4



2 1 2

$$4 \overline{) \begin{array}{cc} 100 & 100 \\ 100 & 100 \\ 100 & 100 \end{array} \begin{array}{c} 10 \\ 10 \\ 10 \end{array} \begin{array}{cc} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{array}}$$

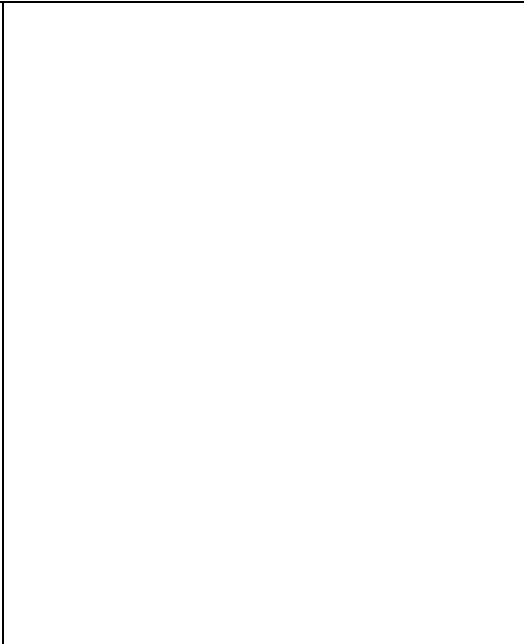
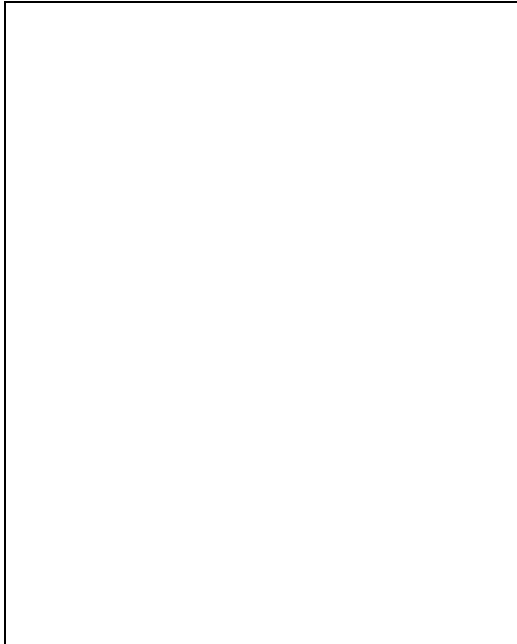
1

$$5 \overline{) \begin{array}{cc} 100 & 100 \\ 100 & 100 \\ 100 & 100 \end{array} \begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array}}$$

1

$$5 \overline{) \begin{array}{cc} 100 & 100 \\ 100 & 100 \\ 100 & 100 \end{array} \begin{array}{cc} 10 & 10 \\ 10 & 10 \\ 10 & 10 \\ 10 & 10 \\ 10 & 10 \\ 10 & 10 \\ 10 & 10 \\ 10 & 10 \end{array} \begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{array}}$$
$$4 \overline{) \begin{array}{r} 212 \\ 848 \end{array}}$$

$$5 \overline{) \begin{array}{r} 141 \\ 7205 \end{array}}$$



$612 \div 4 = 153$

If there is a multiplicative change to the dividend factor and a corresponding change to the divisor, the quotient remains the same.

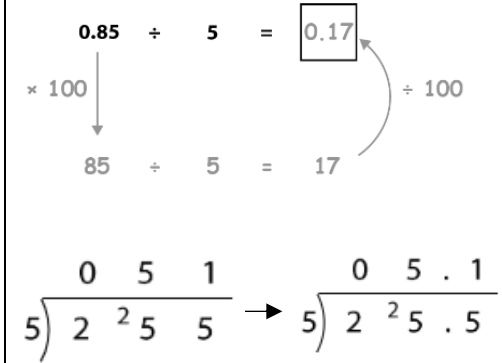
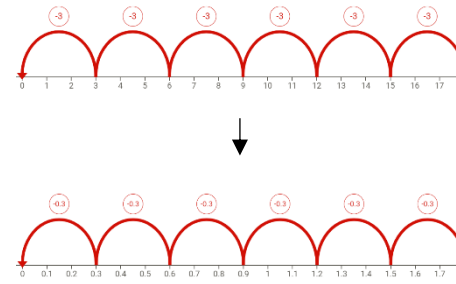
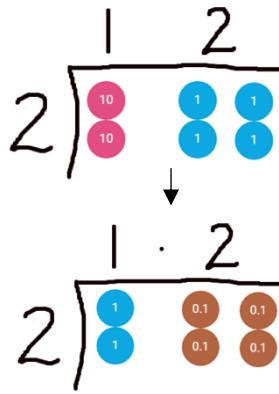
If I multiply the dividend by __, I must multiply the divisor by __ for the quotient to remain the same.

Year 5 and 6

If the dividend is made one tenth of the size, the quotient will be one tenth of the size.

If the dividend is made one hundredth of the size, the quotient will be one hundredth of the size.

I move the digits of the dividend ___ places to the left until I get a whole number; then I divide; then I move the digits of the quotient ___ places to the right.

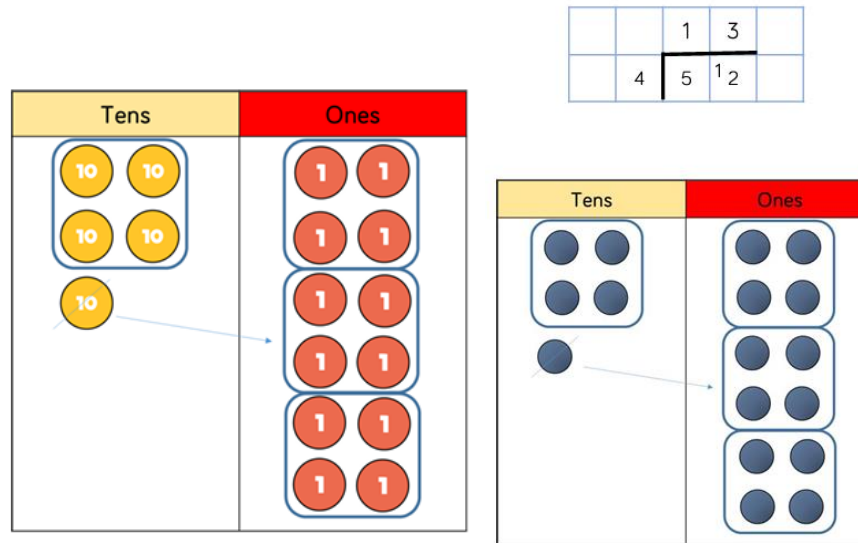


Year 5 onwards

Divide 2-digits by 1-digit (grouping)

When using the short division method, children use grouping. Start with the largest place value, they group by the divisor. Language is important here. Children should consider 'How many groups of 4 tens can we make?' and 'How many groups of 4 ones can we make?'

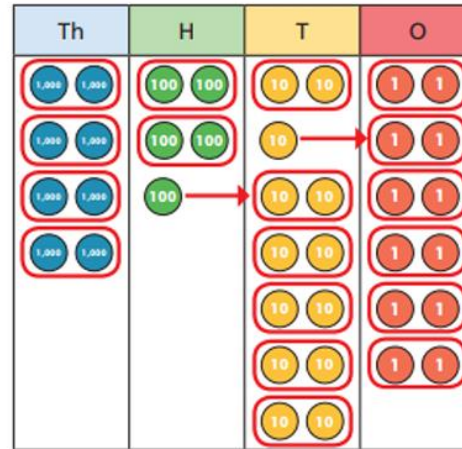
Remainders can also be seen as they are left ungrouped.



Year 5

Divide 4-digits by 1-digit (grouping)

Place value counters or plain counters can be used on a place value grid to support children to divide 4-digits by 1-digit. Children can also draw their own counters and group them through a more pictorial method. Children should be encouraged to move away from the pictorial and concrete when dividing numbers with multiple exchanges.



	4	2	6	6
2	8	5	13	12

Any two-, three- or four-digit dividend can be divided by a two-digit divisor using skip-counting in multiples of the divisor, or by short division or long division.

Where there is a remainder, the result can be expressed as a whole-number quotient with a whole-number remainder, a whole-number quotient with a proper-fraction remainder, or as a decimal-fraction quotient.

Year 6

When children begin to divide up to 4-digits by 2-digits, written methods become the most accurate as concrete and pictorial representations become less effective. Children are encouraged to write out multiples to support their calculations with larger remainders. Children will also solve problems with remainders

Short division

$$\begin{array}{r} 0 \quad 1 \quad 4 \\ 31 \overline{) 4 \quad 43 \quad 124} \end{array}$$

		0	3	6
	12	4	43	72

$$432 \div 12 = 36$$

where the quotient can be rounded as appropriate.

$$7,335 \div 15 = 489$$

	0	4	8	9
15	7	7_3	13_3	13_5

15	30	45	60	75	90	105	120	135	150
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